Propagation of Magnetization of Iron as Affected by the Electric Currents in the Iron

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III. *Propagation of Magnetization of Iron as affected by the Electric Currents in the Iron.*

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PART I.

IT is not unfamiliar to those who have worked on large dynamos with the ballistic galvanometer, that the indications of the galvanometer do not give the whole changes which occur in the induction. Let the deflections of the galvanometer connected to an exploring coil be observed when the main current in the magnetic coils is reversed. The first elongation will be much greater than the second in the other direction, and probably the third greater than the second—showing that a continued current exists in one direction for a time comparable with the time of oscillation of the galvanometer. These effects cannot be got rid of, though they can be diminished by passing the exciting current through a non-inductive resistance and increasing the electromotive force employed. This if carried far enough would be effective if the iron of the cores were divided so that no currents could exist in the iron; but the currents in the iron, if the core is solid, continue for a considerable time and maintain the magnetism of the interior of the core in the direction it had before reversal of current. It was one of our objects to investigate this more closely by ascertaining the changes occurring at different depths in a core in terms of the time after reversal has been made.

The experiments were carried out in the SIEMENS Laboratory, King's College, London; and the electro-magnet used is shown in fig. 1. It consists in its first form, the results of which though instructive are not satisfactory, of two vertical wrought-iron cores, 18 inches long and 4 inches diameter, wound with 2595 and 2613 turns respectively of No. 16 B.W.G. cotton-covered copper wire—the resistance of the two coils in series being 16·3 ohms. The yoke is of wrought-iron 4 inches square in section and 2 feet long. The pole-pieces are of wrought-iron 4 inches square, and all surfaces in contact are truly planed. One of the pole-pieces is turned down at the end, which butts on the other pole-piece, for half an inch of its length to a diameter of 4 inches; and three circular grooves are cut in the abutting face having mean

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* The experimental work of this paper was in part carried out by three of the Student Demonstrators of the SIEMENS Laboratory, King's College, London, Messrs. BRAZIL, ATCHISON, and GREENHAM. We wish to express our thanks to them for their zealous co-operation.

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Fig. 1.

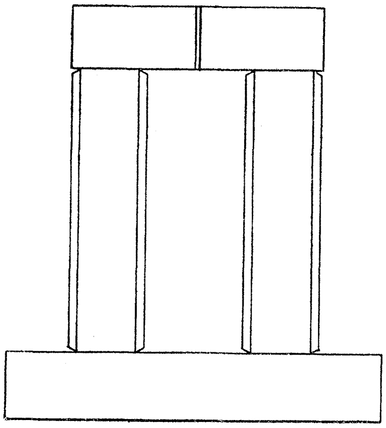


Fig. 3.

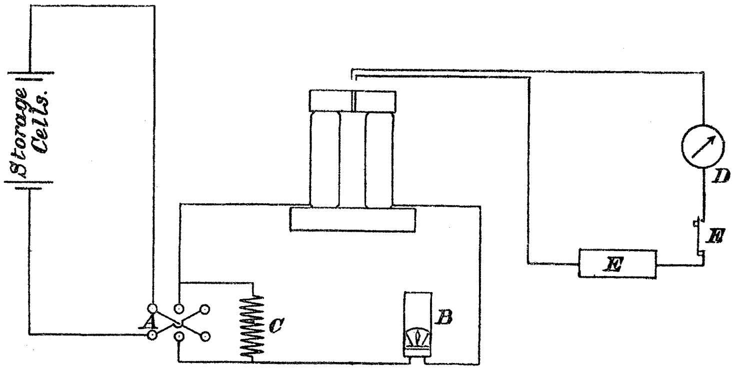


Fig. 2.

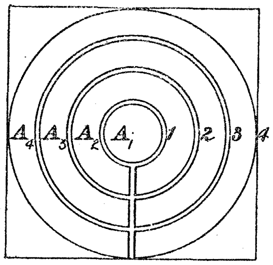


Fig. 5.

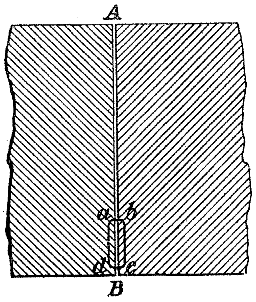


Fig. 6.

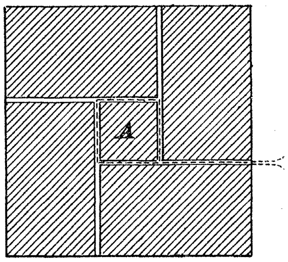


Fig. 4.

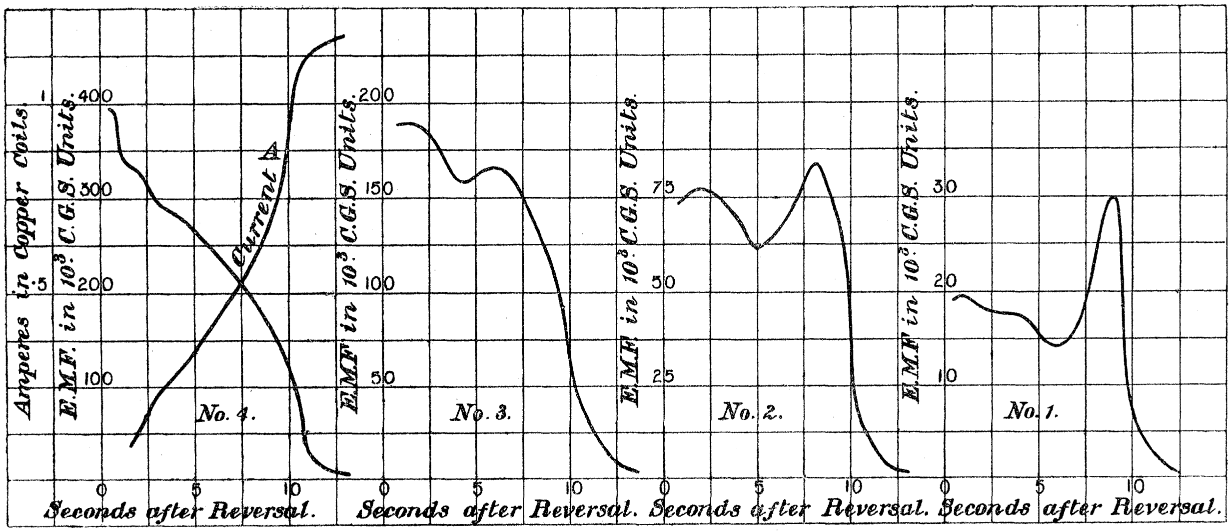


Fig. 7.

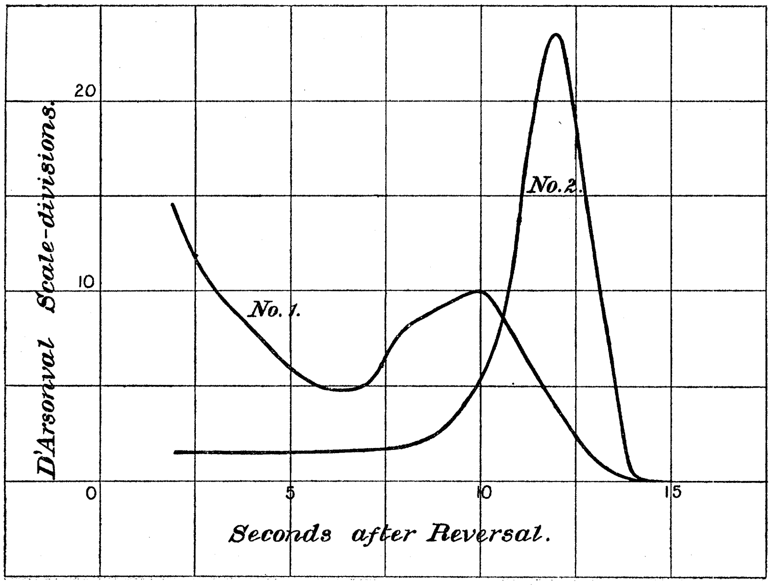


Fig. 9A.

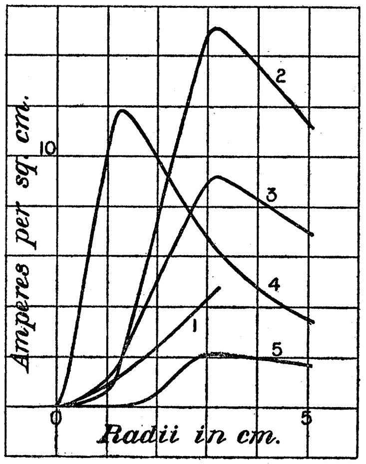


Fig. 9

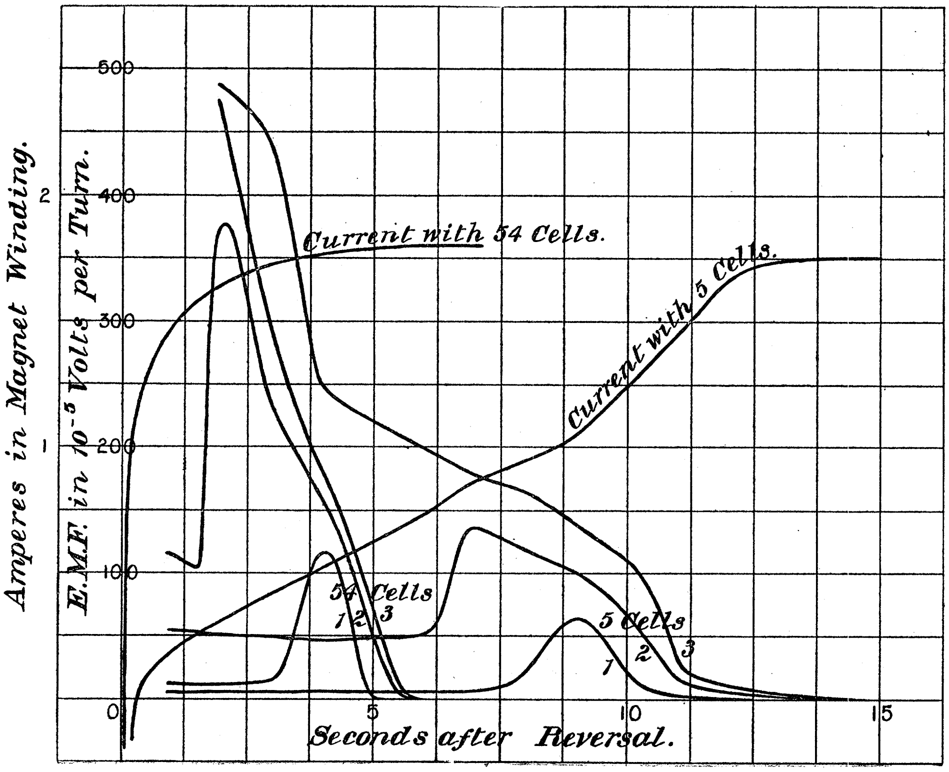
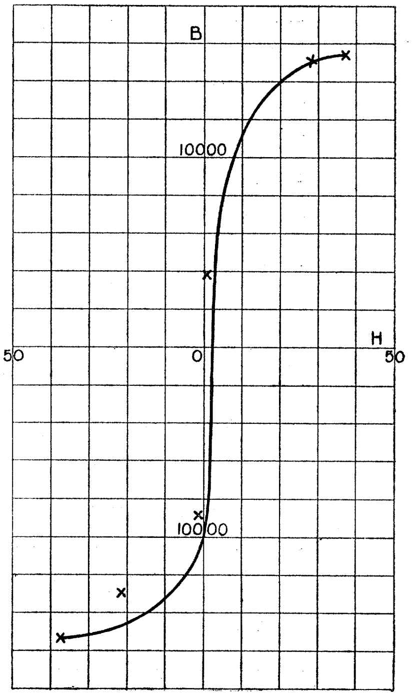


Fig. 9B.



diameters of 2·6, 5·16, and 7·75 centims. respectively, for the purpose of inserting copper coils the ends of which are brought out by means of the radial slot shown in fig. 2. When the pole-pieces are brought into contact as shown in fig. 1, we have thus three exploring coils within the mass and a fourth was wound on the circular portion outside. These exploring coils are numbered 1, 2, 3, 4 respectively, starting with the coil of least diameter.

Fig. 3 gives a diagram of the apparatus and connections, in which A is a reversing switch for the purpose of reversing a current given by ten storage cells through the magnet windings in series; B is a Thomson graded galvanometer for measuring current; and C is a non-inductive resistance of about 16 ohms placed across the magnet coils for the purpose of diminishing the violence of the change on reversal. The maximum current given by the battery was 1·2 amperes. A D'Arsonval galvanometer of Professor AYRTON'S type, D, of 320 ohms resistance; a resistance box E; and a key F were placed in circuit with any one of the exploring coils 1, 2, 3, 4, for the purpose of observing the electromotive force of that circuit. The method of experiment was as follows:—The current round the magnet limbs was suddenly reversed and readings on the D'Arsonval galvanometer were taken on each coil at known epochs after the reversal. The results are shown in fig. 4, in which the ordinates are the electromotive forces in C.G.S. units and the abscissæ are in seconds.

The portion of these curves up to two seconds was obtained by means of a ballistic galvanometer having a periodic time of fifty seconds, the key of its circuit being broken at known epochs after reversal. From the induction curve so obtained the electromotive force was found by differentiation.

The curve A which is superposed on curve 4 of fig. 4 gives the current round the magnet in the magnetizing coils. It is worth noting, that, as would be expected, it agrees with the curve 4. The potential of the battery was 1·2 amperes × 16·3 ohms = 19·6 volts. Take the points two seconds after reversal, the electromotive force in one coil is 330,000; multiplying this by 5208, the number of coils on the magnet, we have in absolute units 1,718,640,000 as the electromotive force on the coil due to electromagnetic change, or, say, 17·2 volts. Subtracting this from 19·6 we have 2·4. The electromotive force observed is ·125 × 16·3 = 2·02. The difference between these could be fully accounted for by an error of $\frac{1}{4}$ second in the time of either observation.

The general character of the results was quite unexpected by us. Take coil No. 2 for example, the spot of light, on reversing the current in the magnet winding, would at once spring off to a considerable deflection, the deflection would presently diminish, attaining a minimum after about 6 seconds; the deflection would then again increase and attain a maximum greater than the first after 8 seconds, it would then diminish and rapidly die away.

To attempt a thorough explanation of the peculiarities of these curves would mean solving the differential equation connecting induction with time and radius in the iron with the true relation of induction and magnetizing force. But we may inversely from these curves attempt to obtain an approximation to the cyclic curve of induction of the iron.

Let l be the mean length of lines of force in the magnet. Let n be the number of convolutions on the magnet, and let c be the current in amperes in the magnetizing coils at time t. Then at this epoch the force due to the magnetizing coils is $4\pi nc/10l$. Call this H_1.

Next consider only one centimetre length of the magnet in the part between the pole-pieces which is circular and has coils 1, 2, 3, wound within its mass, and coil 4 wound outside. The area of each of the electromotive force curves of the coils 1, 2, 3, 4, up to the ordinate corresponding to any time, is equal to the total change of the induction up to that time.

In fig. 2 let A_1, A_2, A_3, A_4 be the areas in sq. centims. of coil 1 and the ring-shaped areas included between the coils 1, 2, 3, 4 respectively. Then the induction at time t, as given by the integral of curve 1, divided by A_1 is the average induction per sq. centim. for this epoch over this area. Also, the induction at time t, as given by the integral of curve 2, *minus* the induction for the same time, as given by the integral of curve 1, divided by A_2, is the average induction per sq. centim. for this area. Similarly, average induction per sq. centim. for A_3, A_4 can be found for any epoch.

Consider area A_1. It is obvious that all currents induced within the mass considered *external* to this area, due to changes of induction, *plus* the current in the magnetizing coil per centim. linear, at any epoch, go to magnetize this area, and, further, the induced currents in the outside of the area A_1 itself go to magnetize the interior portion of this area. We know the electromotive forces at the radii 1, 2, 3, 4, and the lengths in centims. of circles corresponding to these radii. From a knowledge of the specific resistance of the iron we can find the resistance, in ohms, of rings of the iron corresponding to these radii, having a cross-sectional area of 1 sq. centim. Let these resistances be respectively r_1, r_2, r_3, r_4. At time t, let e_1, e_2, e_3, e_4 be the electromotive forces in volts at the radii 1, 2, 3, 4, then $\frac{e_1}{r_1}$, $\frac{e_2}{r_2}$, $\frac{e_3}{r_3}$, $\frac{e_4}{r_4}$ are at this epoch the amperes per sq. centim. at these radii. Let a curve be drawn for this epoch, having amperes per sq. centim. for ordinates and radii in centims. for abscissæ. Then the area of this curve, from radius 1 to radius 4, gives approximately the amperes per centim. due to changes of induction, and (neglecting the currents within the area considered) the algebraic sum of this force (call it H_2), with the force due to the magnetizing coils (H_1) at the epoch chosen, gives the resultant magnetizing force acting upon area A_1. If H is this resultant force, we have $H = H_1 + H_2$. Next draw a curve showing the relation between the induction per sq. centim. (B) and the resultant force (H) for different epochs. This curve should be an approximation to the cyclic curve of induction of the iron.

Fig. 10.

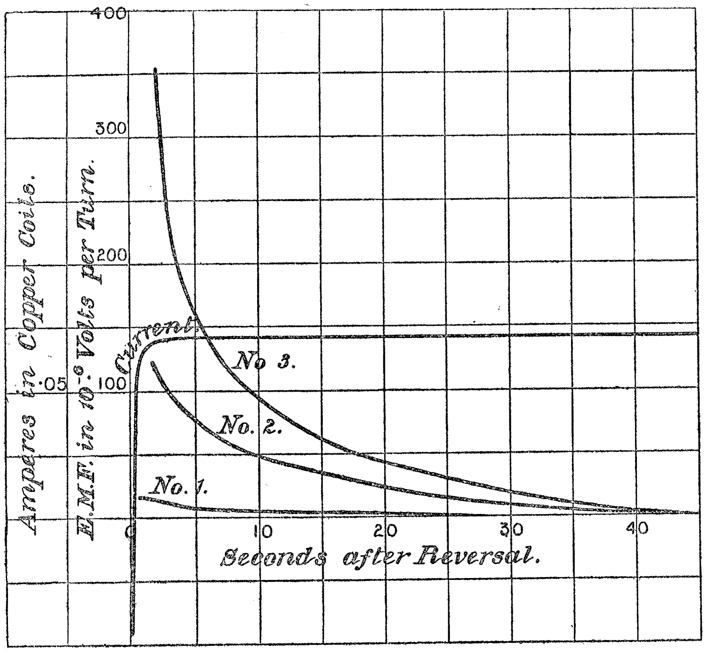


Fig. 13.

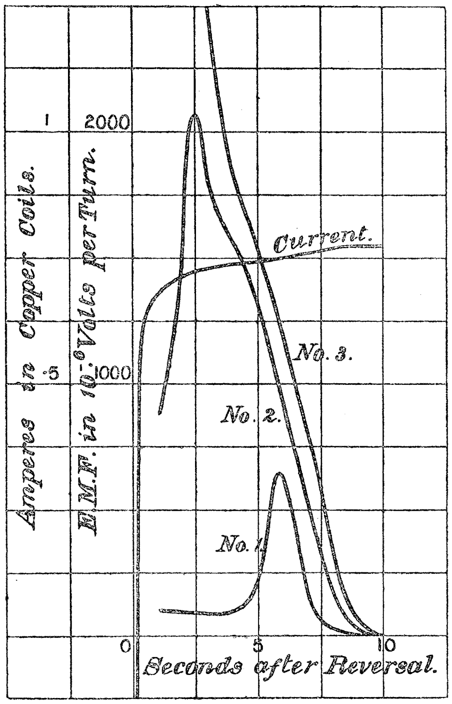


Fig. 13A.

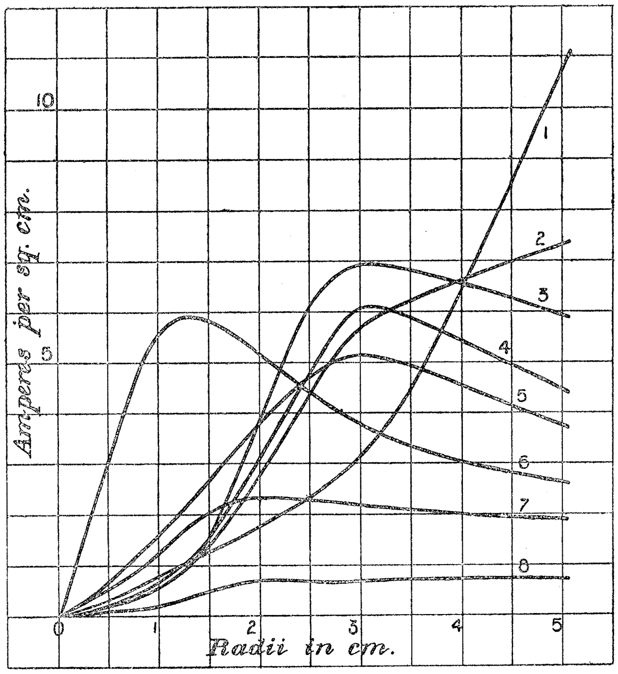


Fig. 14.

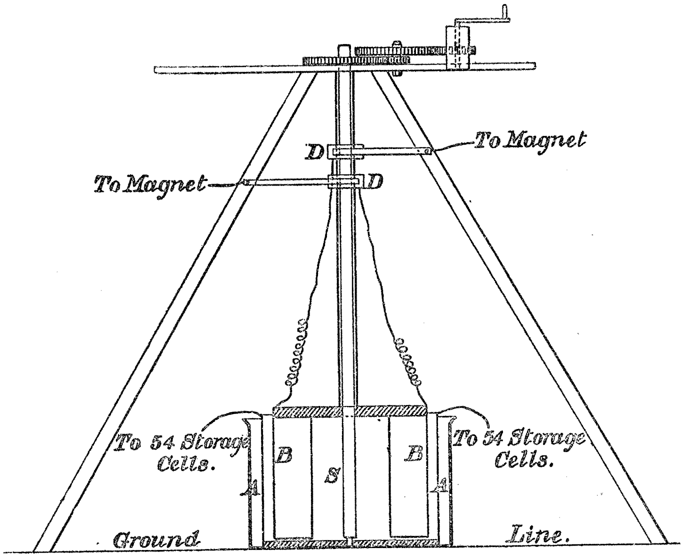


Fig. 15.

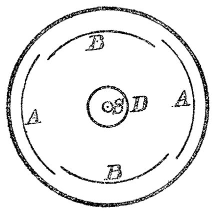


Fig. 16.

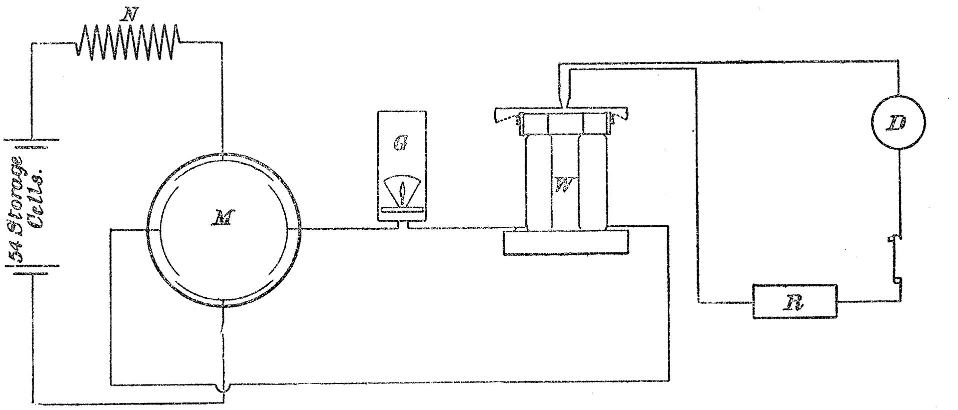


Fig. 11.

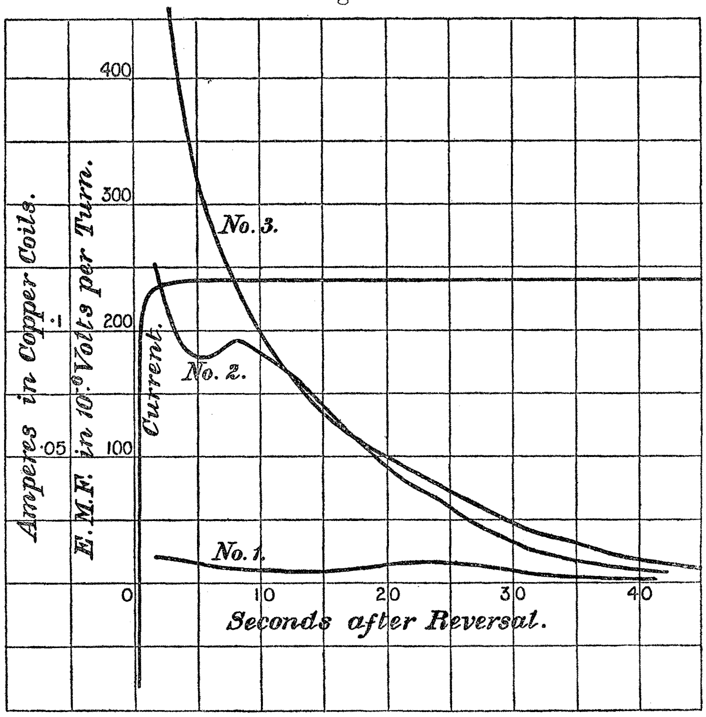


Fig. 12A.

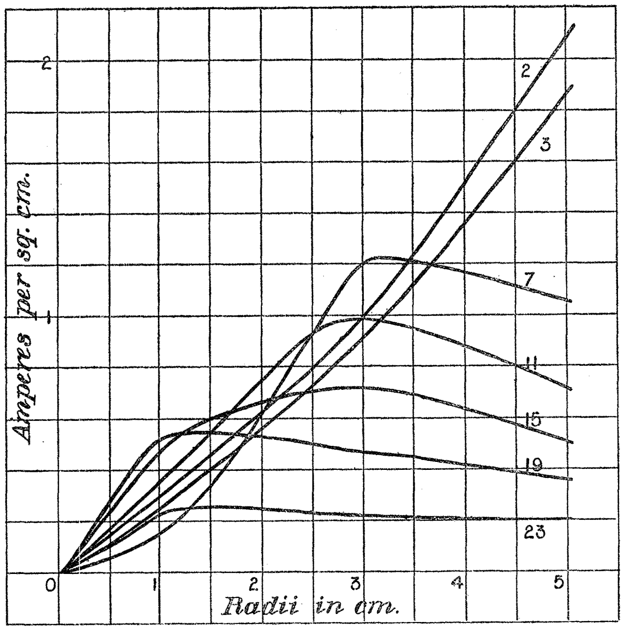


Fig. 12.

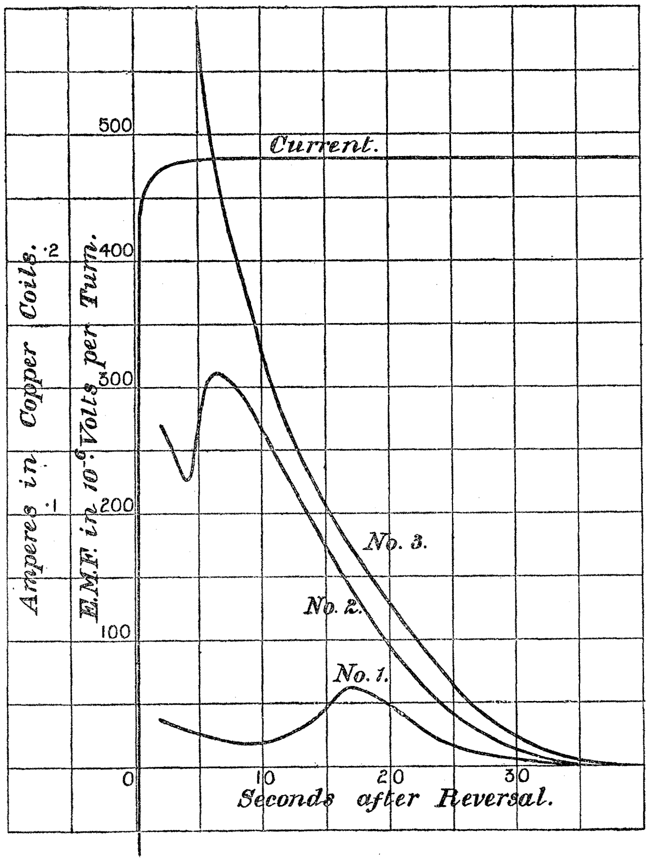
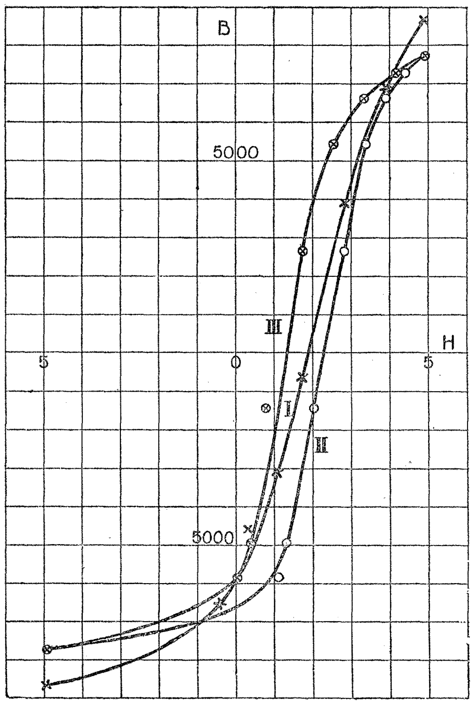


Fig. 12B.



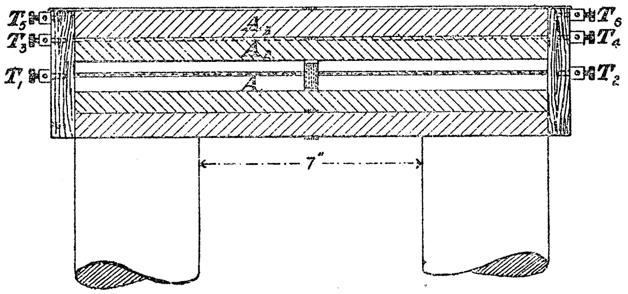
The attempt to obtain an approximation to the cyclic curve of induction from the curves in fig. 4 was a failure, that is to say, the resulting curve did not resemble a cyclic curve of magnetization. This is due to imperfections of fit of the two faces, in one of which the exploring coils are imbedded. That this imperfection of fit will tend to have a serious effect upon the distribution of induction over the whole area is obvious on consideration. Take the closed curve *abcd* in fig. 5, where AB is the junction between the pole pieces. If the space between the faces was appreciable, the force along *bc* and *ad* in the iron could be neglected in comparison with the forces in the non-magnetic spaces *ab*, *cd*. The magnetizing force is sensibly $4\pi c$, where c is the current passing through the closed curve. This may be made as small as we please. Therefore, the force along *ab* is equal to the force along *dc*. In our case the space between the faces is very small, but has still a tendency towards an equalizing of the induction per unit area over the whole surface.

To test this the following experiment was tried. At a distance of $2\frac{1}{2}$ inches from the abutting surfaces of the pole pieces four holes were drilled in one of the pole pieces in a plane parallel with the abutting surfaces, as shown in fig. 6. By means of a hooked wire we were able to thread an insulated copper wire through these holes, so as to enclose only the square area A, which is bounded by the drilled holes and has an area of ·61 sq. inch. The wire is indicated by the dotted lines. Fig. 7 gives two curves taken by the D'Arsonval in the manner already described for a reversal of the same current in the copper coils of the magnets. No. 1 (fig. 7) is the curve obtained from No. 1 coil (fig. 2) near the air space. No. 2 (fig. 7) is the curve obtained from the square coil shown in fig. 6. The difference is very marked and shows at once the effect of the small non-magnetic space which accounts for the large initial change of induction previously observed on the coils 1, 2, 3 in fig. 4. Similar holes were drilled in the yoke of the magnet in a plane midway between the vertical cores, having the same area of ·61 sq. inch; and on trial exactly the same form of curve was produced as is shown in No. 2 of fig. 7. This method of drilling holes in the mass is open to the objection that the form of the area is square.

Whilst the above experiments were being made the portion of the magnet to take the place of the pole-pieces previously used was being constructed as follows:—In fig. 8 the portion of the magnetic circuit resting upon the vertical cores consists of a centre rod A_1 of very soft Whitworth steel surrounded by tubes A_2, A_3 of the same material. The diameter of A_1 is 1 inch. The outside diameter of A_2 is $2\frac{1}{2}$ inches; and A_3 is 4 inches outside diameter between the cores of the magnet, but is 4 inches square at each end where it rests upon the magnet limbs. At the centre of the rod A_1 (longitudinally) a circular groove is turned down 1 millim. deep and 5 millims. wide, and also a longitudinal groove 1 millim. deep and 1 millim. wide is cut as shown in the figure for the purpose of leading a double silk covered copper wire from terminal T_1 to 9 convolutions at the centre and along the rod to terminal T_2. A similar groove is cut in the outside of the tube A_2, and a copper wire is carried from terminal T_3 to 9 convolutions round the centre of the tube again along the groove to terminal T_4. Nine convolutions were also wound round the outside tube A_3, the ends of which are connected to the terminals T_5, T_6 respectively.

The tubes and rod were made by Sir J. WHITWORTH and Co., of Manchester, and a considerable force was required to drive the pieces into their proper position. Our best thanks are due to Professor KENNEDY and his assistants for the putting together of these pieces by means of a 50-ton hydraulic testing machine. We are aware that the surfaces are somewhat scored by the hydraulic pressure, and the magnetic qualities may be slightly different for layers of the soft steel near these surfaces, but they serve just as well for the purpose of our experiments.

Fig. 8.



Systematic experiments were then commenced. The magnetizing coils on the magnets were placed in parallel with one another, and a total current of 1·75 amperes (that is, ·87 ampere in each coil), due to 5 storage cells, was reversed through the coils. The arrangement of apparatus is shown in fig. 3, except that the pole-pieces are replaced by the soft steel tubes shown in fig. 8, and the non-inductive resistance C is removed. We have now three exploring coils instead of four, and these are marked 1, 2, 3 respectively, starting with the coil of smallest diameter. For the purpose of obtaining the current curve, the D'Arsonval was placed across a non-inductive resistance of $\frac{1}{8}$ ohm in the circuit of the magnetizing coils. Fig. 9 gives a set of curves obtained with the 5 cells, and also another set obtained by a reversal of 1·8 amperes given by 54 cells—a non-inductive resistance being placed in the circuit to adjust the current.

The effect of reversing the same maximum current with two different potentials is very marked. Take coil No. 1. With 5 cells the maximum rate of change of induction occurs at 9 seconds after reversal, at which epoch the current in the copper coils is about 1 ampere, the maximum current being 1·75. With 54 cells the maximum rate of change of induction occurs at 4 seconds, and here the current in the copper coils is nearly a maximum. We therefore chose to work with 54 cells, thus avoiding a magnetizing force due to the current in the copper coils varying for considerable times after reversal.

Fig. 17.

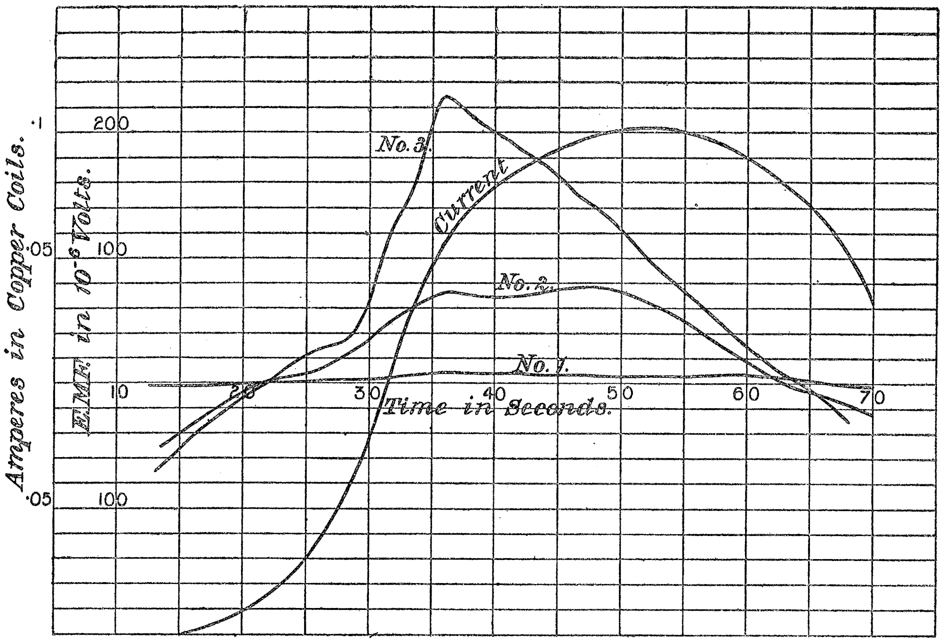


Fig. 18.

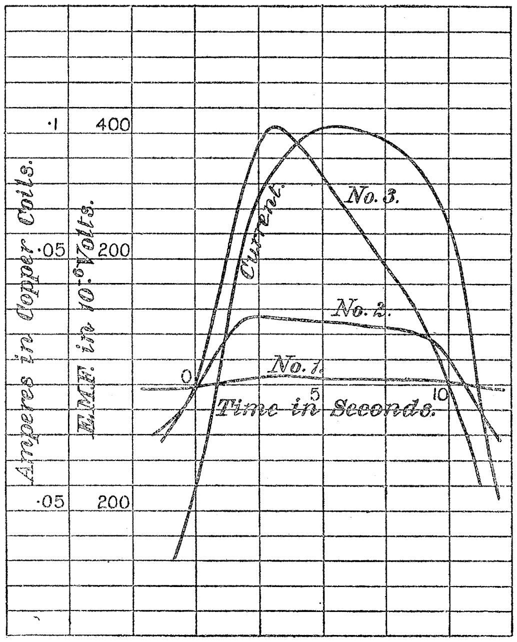


Fig. 19.

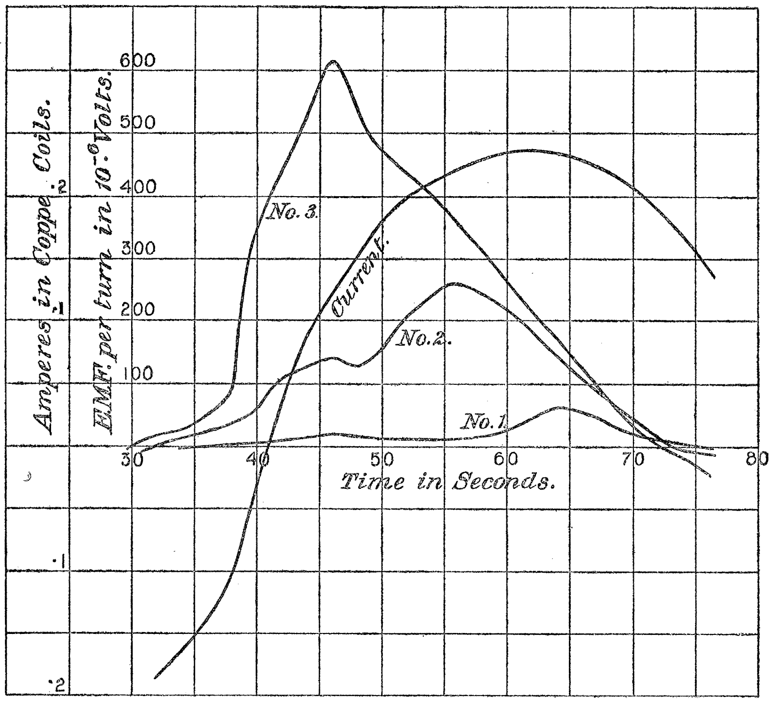


Fig. 19A.

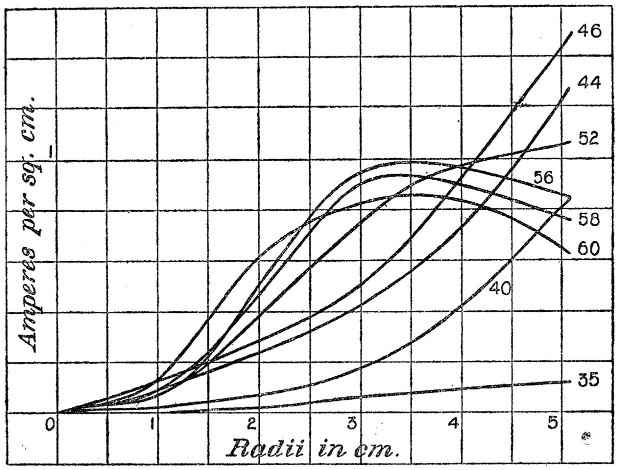


Fig. 19A.—continued.

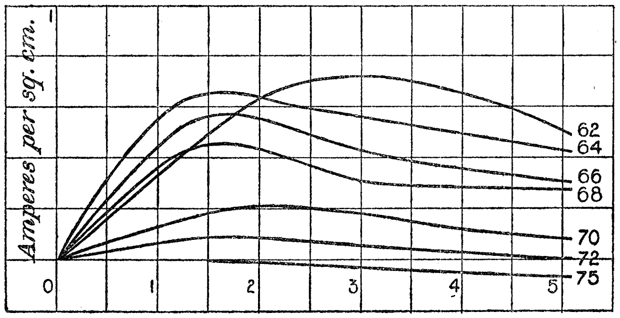


Fig. 19B.

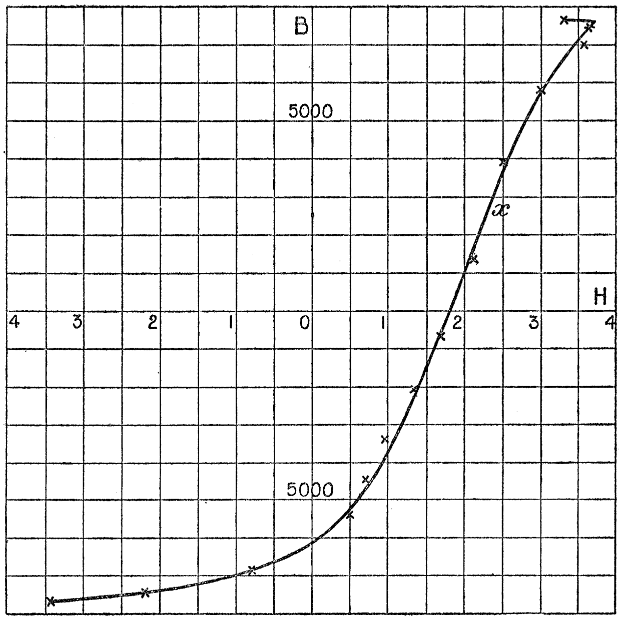


Fig. 19C.

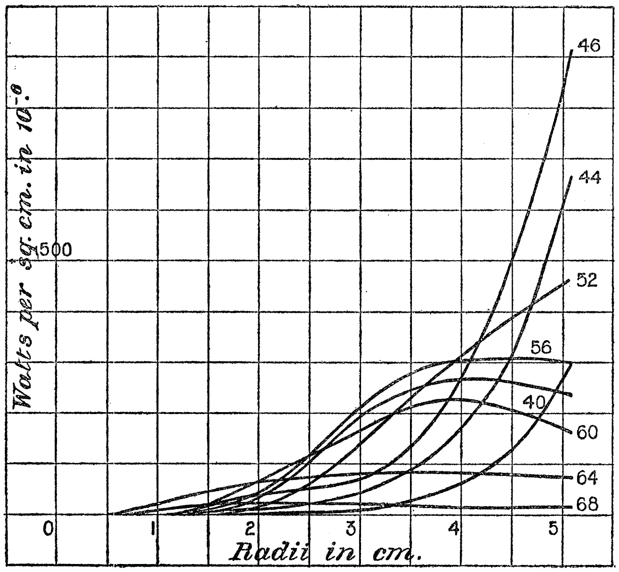


Fig. 19D.

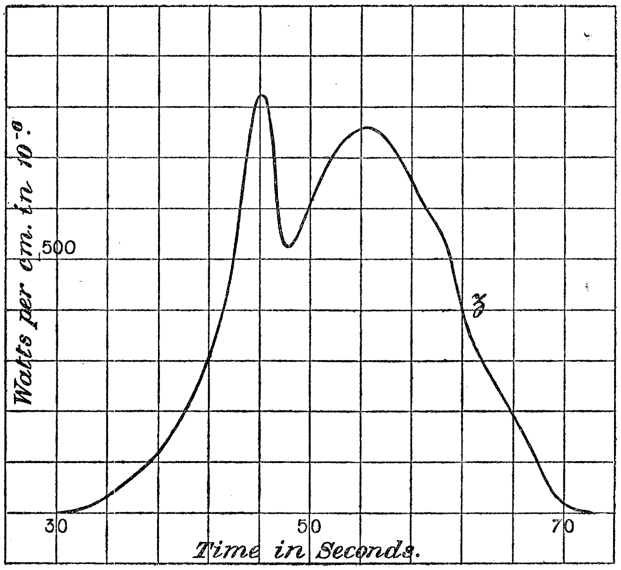


Table I. gives a list of the experiments made with total reversal of current due to 54 cells, the magnetizing coils being kept in parallel with one another, and the magnitude of current through them adjusted by means of a non-inductive resistance.

In fig. 10 the maximum current in the copper coils is ·0745 ampere, which, after reversal, passes through zero and attains a maximum at about 3 seconds. It will be observed that the change of induction with regard to each of the coils 1, 2, 3 is rapid to begin with, but that it gradually decays and becomes zero at about 46 seconds after reversal.

Fig. 11 is interesting in that it gives the particular force at which coils 1 and 2 show a *second* rise in the electromotive force curves, No. 1 being a maximum at about 25 seconds, and No. 2 at about 8 seconds after reversal. These "humps" become a flat on the curve for a little smaller force, and, as shown in fig. 10, they have disappeared altogether. In this case the current in the copper coils has attained a maximum at about 4 seconds after reversal.

In fig. 12 the maximum current in the copper coils is ·24 ampere, corresponding with a force in C.G.S. units of 4·96. This is got from $\frac{4\pi}{10} \frac{2600 \times \cdot 24}{158}$. The current in the copper coils has attained its maximum value at about 4 seconds after reversal, and changes of induction were going on up to 35 seconds.

In the following attempt to obtain an approximation to the cyclic curve of hysteresis, from these curves, we have taken the volume-specific resistance of the soft steel to be 13×10^{-6} ohm. We have taken the diameter of coils 1, 2, 3 to be respectively 1·22, 3·18, and 5·08 centims.,* and we find that the corresponding resistances, in ohms, of rings of the steel having 1 sq. centim. cross-section and mean diameters equal to the coils are, respectively, $103{\cdot}7 \times 10^{-6}$, $259{\cdot}4 \times 10^{-6}$, and $416{\cdot}4 \times 10^{-6}$. From a knowledge of the electromotive forces at the three radii, for a given epoch, we are able to find the amperes per sq. centim. at those radii. In fig. 12A a series of curves have been drawn for different epochs, giving the relation between amperes per sq. centim. and radii in centims., and the areas of these curves between different limits have been found, and are tabulated in Table II. It is necessary here to state that the path of these curves through the four given points in each case is assumed; we have simply drawn a fair curve through the points. But what we wish to show is that the results obtained with the curves, drawn as shown in fig. 12A, are not inconsistent with what we know with great probability to be true.

The results shown in fig. 12B have been obtained as follows: take curve 1, fig. 12B; the electromotive force curve of coil 1, fig. 12, has been integrated, and the integral up to the ordinate corresponding to any time is equal to the total

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* In Part II. of this paper the smallest radius was taken to be 1·27. For our purpose the difference is not worth the expense of correction.

change of the induction up to that time, which divided by the area of the coil in sq. centims. gives the average induction per sq. centim. In obtaining the areas we had to assume the path of the electromotive force curve up to 2 seconds, but this we can do with a good deal of certainty.

With regard to the forces we see that after 3 seconds the induced currents have to work against a constant current in the copper coils. In obtaining the forces due to induced currents we have only taken the area of the curves in fig. 12A between the radii 1·22 centims. and 5·08 centims.; that is, we have neglected the effect of the currents within the area of coil No. 1 altogether. The resultant force (H) is the algebraic sum of the force (H_2) due to the currents between the radii taken, and the force (H_1) due to the current in the copper coils, and is set forth for different epochs in Table II. The inductions per sq. centim. have been plotted in terms of this resultant force (H), and curve I., fig. 12B, shows this relation.

Next, take curves II. and III., fig. 12B. In obtaining the inductions for these curves, the difference between the integrals of curves No. 1 and 2, fig. 12, for a given epoch, has been taken. This gives the induction for this epoch, which, when divided by the ring-shaped area between coils 1 and 2, gives the average induction per unit of that area.

In obtaining the forces in curve II., fig. 12B, we have taken the areas of the curves in fig. 12A between the radii 3·18 centims. and 5·08 centims.; that is, we have neglected the forces within the area under consideration as before. Here the error is of more importance, and may partly account for the difference between the forces of curves I., II. In curve III. we have taken the areas of curves in fig. 12A between the radii 2·2 and 5·08; that is, we have taken account of the force due to induced currents over a considerable portion of the area considered. Coupled with the uncertainty in form of the curves in fig. 12A we have the uncertainty as to how much to allow for the forces due to induced currents over the particular area considered. The difference in the ordinates of curves I. and II. may partly be accounted for by errors arising from the assumed path of the electromotive force curve up to 2 seconds, which is more uncertain in curve 2, fig. 12, than in curve 1; and partly to possible slight inequality between the materials of the rod and its surrounding tube.

In fig. 13 the maximum current in the copper coils is ·77 ampere, corresponding with a force in C.G.S. units of 16. The current in the copper coils, after passing through zero, attains its full value at about 9 seconds after reversal, and the change of induction ceases at 10 seconds.

No. 1 curve, fig. 13, has been integrated, and the maximum induction per sq. centim. found to be 14,500 C.G.S. units. We have taken a given cyclic curve for soft iron corresponding with this maximum induction, and have tabulated the forces obtained therefrom in Table III. for the different values of B got from the integration of No. 1 curve. We then plotted in fig. 13A the amperes per sq. centim.

Fig. 20.

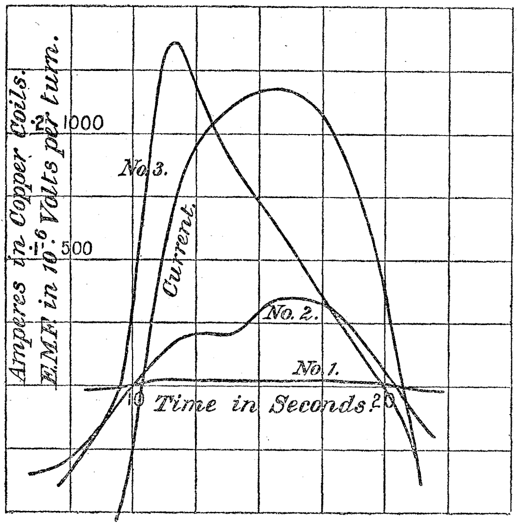


Fig. 20B.

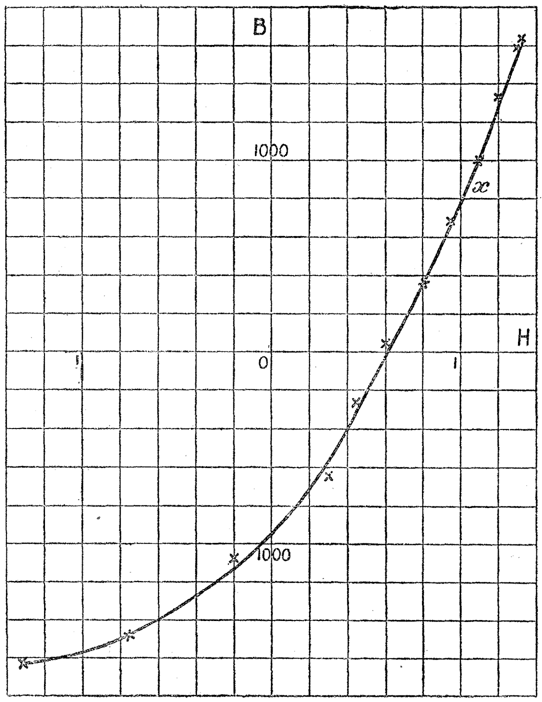


Fig. 20A.

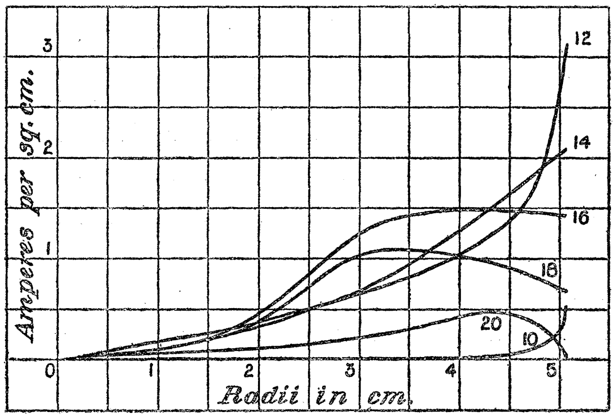


Fig. 20C.

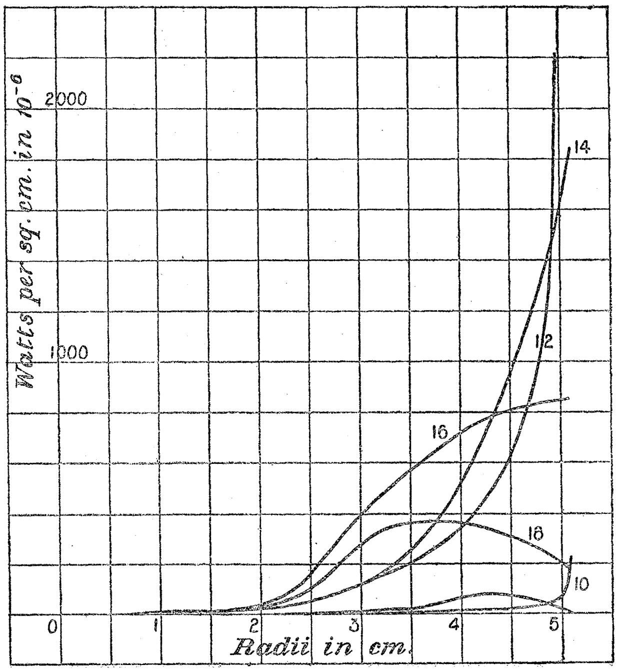


Fig. 20D.

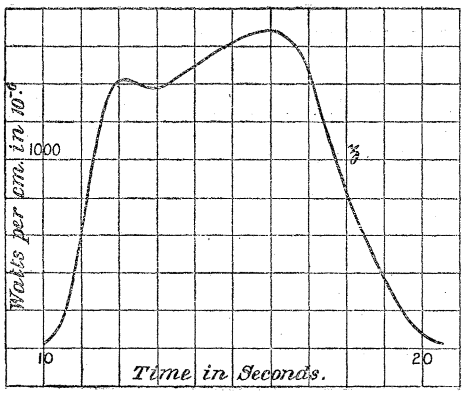


Fig. 21.

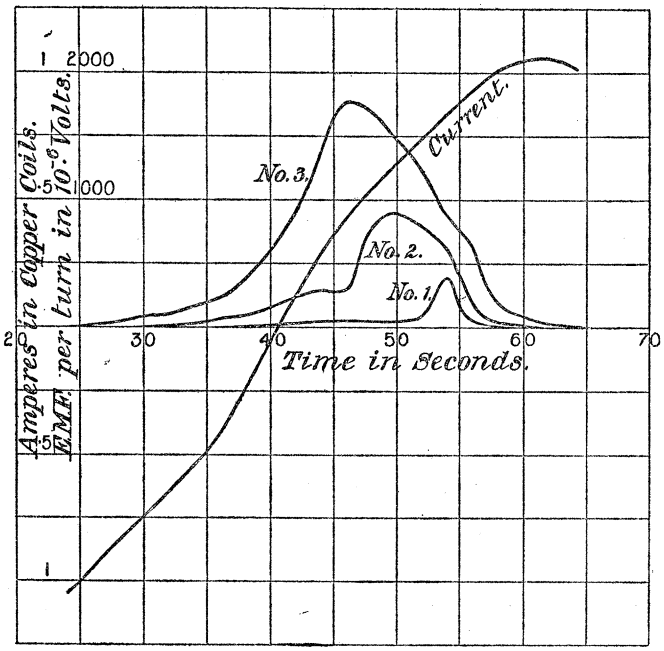


Fig. 21A.

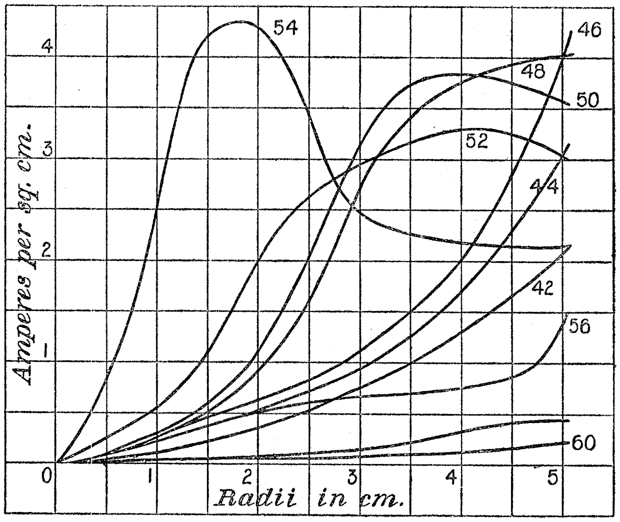


Fig. 21B.

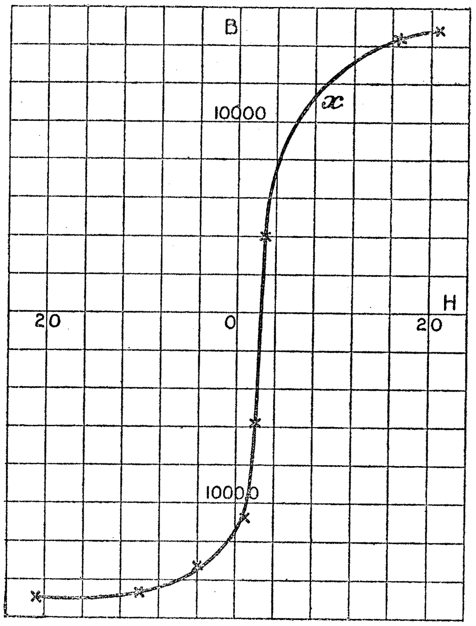


Fig. 21C.

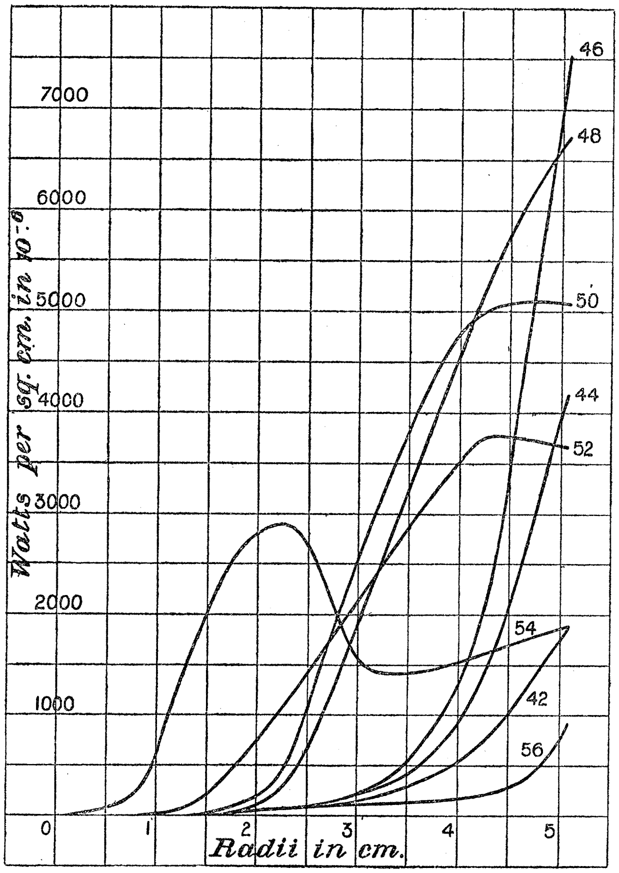
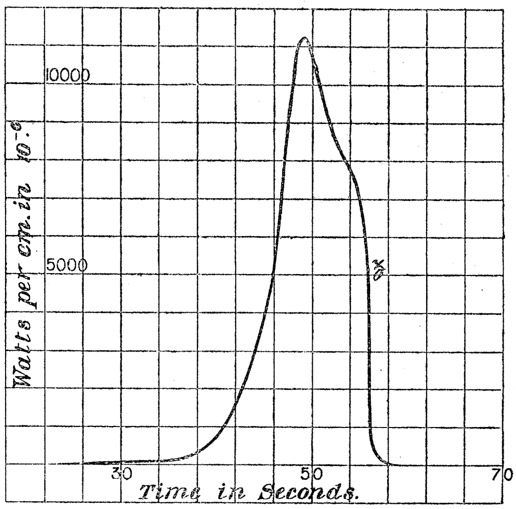


Fig. 21D.



at the different radii for different epochs, and in each case, by drawing a curve fairly through them, we were able to produce areas in fair correspondence with areas as got by means of the given cyclic curve. The comparative areas are tabulated in Table III.

In fig. 9 the maximum current in the copper coils due to the 54 cells is 1·8 amperes, corresponding with a force of 20·7 in C.G.S. units. In this case the current had passed through zero and attained a maximum at 6 seconds after reversal; the change of induction being zero also at this time. We have worked out the current per sq. centim. for the different radii at different epochs, as before, and have plotted them in fig. 9A. Fig 9B gives the relation of B to H, found from the curves, and it also shows a fair approximation to the cyclic curve for soft iron, although in this case the points are fewer in number and were more difficult to obtain, owing to the greater rapidity with which the D'Arsonval needle moved as compared with the earlier curves.

With a reversal of 2·3 amperes the whole induction effects had died out at 5 seconds after reversal. Coil No. 1 showed a maximum electromotive force at about $3\frac{1}{2}$ seconds. Coil No. 2 gave a dwell, and attained a maximum at 2 seconds, and then died rapidly away. Coil No. 3 attained an immediate maximum and died rapidly to zero at 5 seconds.

With a reversal of $6\frac{1}{2}$ amperes the whole inductive effects had died out at about 3 seconds after reversal. No. 1 coil showed a maximum electromotive force at about $1\frac{3}{4}$ seconds. No. 2 gave a dwell and attained a maximum at about $1\frac{1}{2}$ seconds and rapidly died away to zero at about 2 seconds. No. 3 attained an immediate maximum and died rapidly to zero at about 2 seconds.

The variations in form of these curves and of the times the electromotive forces take to die away are intimately connected with the curve of magnetization of the material. When the magnetizing force is small (1·7) the maxima occur early because the ratio induction to magnetizing force is small. As the magnetizing force increases to 3 and 4·96 the maxima occur later because this ratio has increased, whilst when the force is further increased to 16 and 37·2, as shown in figs. 13 and 9, the maxima occur earlier because the ratio has again diminished.

The results, both of these experiments and of those which follow, have a more general application than to bars of the particular size used. From the dimensions of the partial differential equation which expresses the propagation of induction in the bar, one sees at once that if the external magnetizing forces are the same in two bars differing in diameter, then similar magnetic events will occur in the two bars, but at times varying as the square of the diameters of the bars. But one may see this equally without referring to the differential equation. Suppose two bars, one n times the diameter of the other, in which there are equal variations of the magnetizing forces; consider the annulus between radii r_1, r_2 and nr_1, nr_2 in the two, the resistance per centimetre length of the rods of these annuli will be the same for their area, and their lengths are alike as $1 : n$; the inductions through them, when the inductions per centimetre are the same, are as the areas, that is, as $1 : n^2$. Hence if the inductions change at rates inversely proportional to $1 : n^2$, the currents between corresponding radii will be the same at times in the ratio of $1 : n^2$, and the magnetizing forces will also be the same.

Magnets of sixteen inches diameter are not uncommon; with such a magnet, the magnetizing force being 37 and the magnetizing current being compelled to at once attain its full value, it will take over a minute for the centre of the iron to attain its full inductive value.

On the other hand, with a wire or bundle of wires, each 1 millim. diameter, and a magnetizing force between 3 and 5, which gives the longest times with our bar, the centre of the wire will be experiencing its greatest rate of change in about $\frac{1}{500}$ second. This is a magnetizing force similar to those used in transformers, and naturally leads us to the second part of our experiments.

Part II.—Alternate Currents.

This part of the subject has a practical bearing in the case of alternate current transformer cores, and the armature cores of dynamo-electric machines.

The alternate currents used have periodic times, varying from 4 to 80 seconds, and were obtained from a battery of 54 storage cells by means of a liquid reverser,* shown in elevation and plan in figs. 14 and 15. It consists of two upright curved plates of sheet copper, AA, between which were rotated two similar plates, BB, connected with collecting rings, DD, from which the current was led away by brushes to the primary circuit of the magnet. The copper plates are placed in a weak solution of copper sulphate in a porcelain jar. The inner copper plates, and the collecting rings, are fixed to a vertical shaft, S, which can be rotated at any desired speed by means of the gearing shown in the figure. The outer plates are connected to the terminals of the battery of storage cells, and the arrangement gives approximately a sine curve of current when working through a non-inductive resistance.

The experiments were made with the same electro-magnet and Whitworth steel tubes described in Part I. of this paper. Fig. 16 gives a diagram of connections in which M is the current reverser, G is the Thomson graded current meter for measuring the maximum current in the copper coils, and W is the electro-magnet. A small, non-inductive resistance, placed in the primary circuit served to give the curve of current by observations on the D'Arsonval galvanometer, D, of the time variation of the potential difference between its ends. The D'Arsonval galvanometer was also used, as in Part I., for observing the electromotive forces of the exploring coils 1, 2, and 3 (see fig. 8, Part I.), R being an adjustable resistance in its circuit for the purpose of keeping the deflections on the scale.

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* This form of reverser is due to Professor Ewing.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Fig. 22.

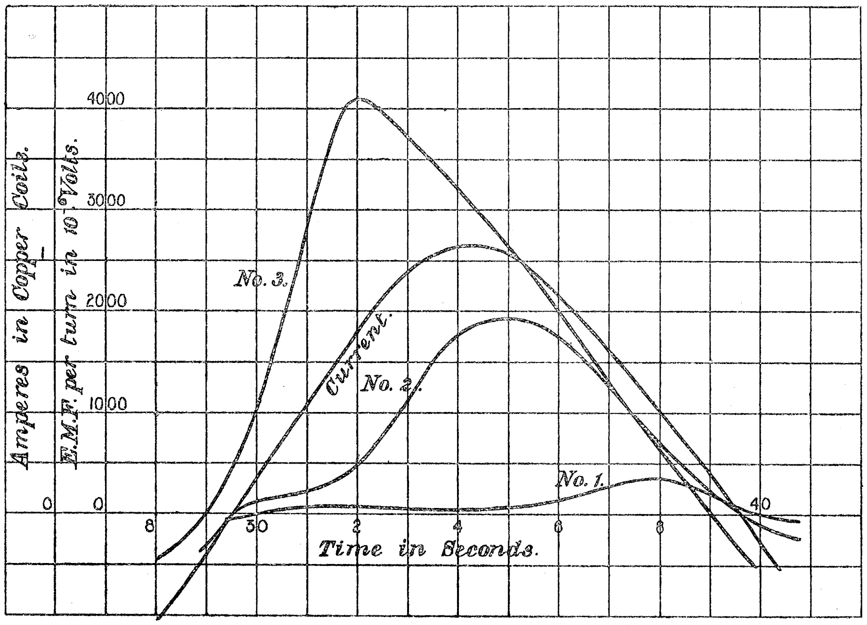


Fig. 22A.

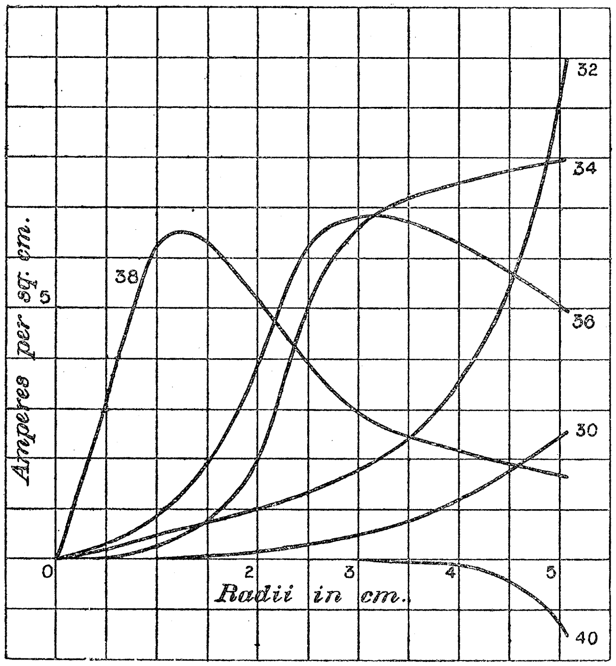


Fig. 22B.

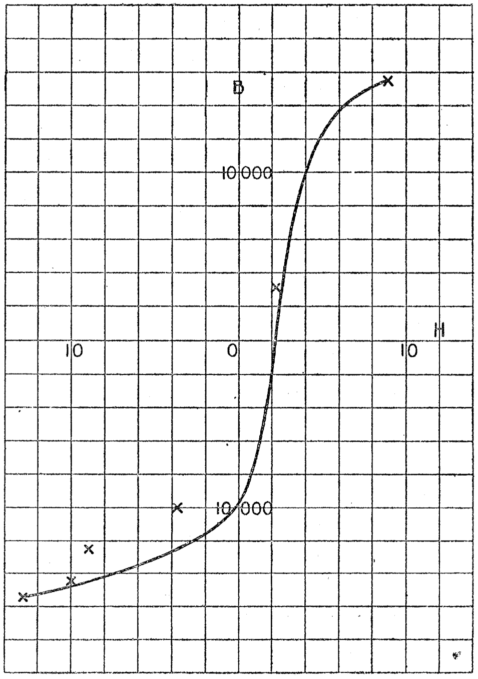


Fig. 22D.

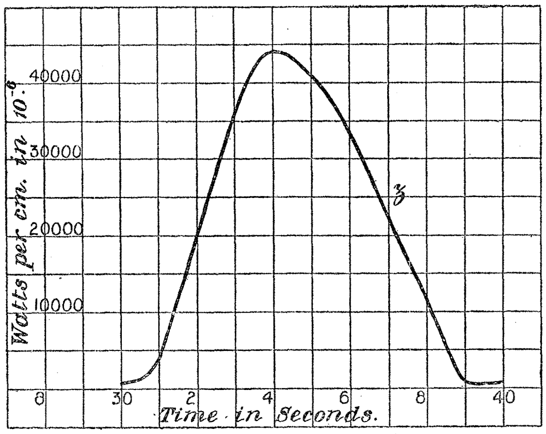


Fig. 22C.

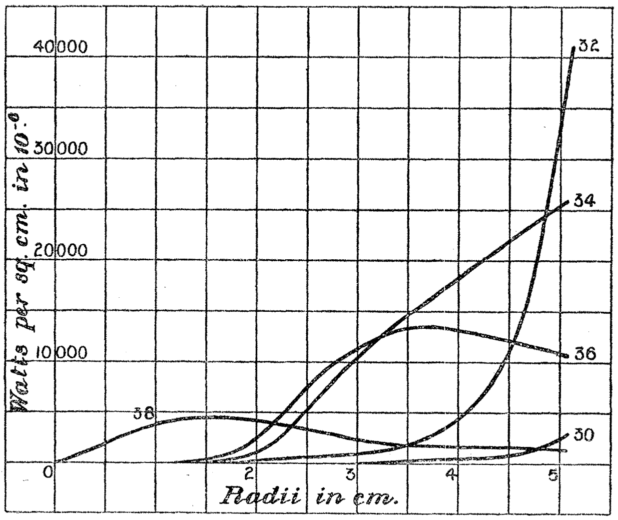
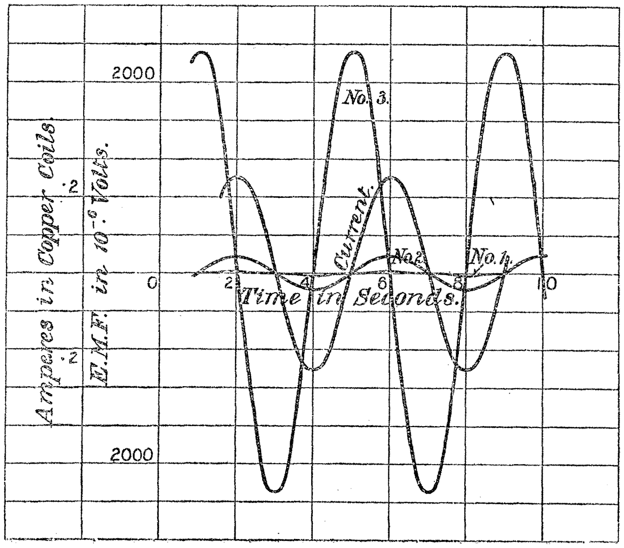


Fig. 23.



The method of experiment was as follows :—The liquid reverser, M, was placed so as to give a maximum current on the meter G, which was adjusted by non-inductive resistance, N, to the desired value, and, in all cases, when changing from higher to lower currents, a system of demagnetization by reversals was adopted. Time was taken, as in Part I., on a clock beating seconds, which could be heard distinctly.

As an example, take fig. 19, in which the periodic time is 80 seconds, and the maximum current in the copper coils ·23 ampere. The E.M.F. curves of the exploring coils are numbered 1, 2, and 3 respectively, and the curve of current in the copper coils is also given.

As in the case of simple reversals (Part I.) we may from these curves attempt to obtain an approximation to the cyclic curve of induction of the iron. In all cases where this is done we have taken coil 1 and considered the area within it—that is to say, from a knowledge of the E.M.F.'s at different depths of the iron, due to change of induction at any epoch, we have estimated the average magnetizing force acting in this area, and this we call H_2. The curves from which these forces have been obtained are given in fig. 19A, and have been plotted from Table VI. The algebraic sum of this force, H_2, and the force H_1, given at the same epoch by the current in the copper coils, is taken to be the *then* resultant force magnetizing this area. Also the integral of curve 1, fig. 19, gives the average induction over this area at the same epoch. Curve x, fig. 19B, is the cyclic curve obtained by plotting the inductions in terms of the resultant force H.

A word is necessary with regard to the last column in Table VI. This gives the total dissipation of energy by induced currents in ergs per cycle per cub. centim. of the iron. We know the watts per sq. centim. at different depths of the iron for different epochs. Let a series of curves be drawn (fig. 19C) for chosen epochs giving this relation : the areas of these curves from radii 0 to 5·08 give for the respective epochs the watts per centim. dissipated by induced currents. In symbols this is $\int \frac{ec}{\text{sq. centim.}}\, dr$; where r is the radius, and ec the E.M.F. and current. It is now only necessary to integrate with regard to time in order to obtain the total dissipation : we have chosen a half period as our limits. This gives us $\iint \frac{ec}{\text{sq. centim.}}\, dr\, dt$, and is got from the area of curve z, fig. 19D. The ordinates of this curve are taken from the last column of Table VI.

The curves in figs. 21, 22, have been treated in a similar manner to that already described in connection with fig. 19. But in fig. 20 the procedure is a little different. In this case the periodic time is 20, and the maximum force per centim. linear, due to the current in the copper coils, is 4·87. With this frequency and current the effects of induced currents in the iron are very marked : we have taken a given soft iron cyclic curve, of roughly the same maximum induction as given by the integral of curve No. 1, fig. 20, and have tabulated the forces obtained therefrom in Table VII.

In fig. 20A we have plotted the amperes per sq. centim. at the different radii, and for the several epochs, and in each case, by drawing a curve fairly through these points, as shown in the figure, we are able to produce areas in fair correspondence with the areas obtained by means of the given cyclic curve. The comparative areas are given in Table VII.

The results shown in fig. 22 are by no means so satisfactory as the results given by other figures, but we have thought it better to insert them here, as we do not wish to make any selection of results which might give an idea of average accuracy greater than these experiments are entitled to.

Referring now to the summary of results in Table V., we note the marked effect of change of frequency upon the average induction per unit area of the innermost coil No. 1, when dealing with comparatively small maximum inductions. Compare the results given in figs. 19 and 20. The maximum force per centim. linear due to the current in the copper coils is 4·8 in each case, but the average induction per sq. centim. of coil No. 1 is reduced from 7690 to 1630 by a change of frequency from $\frac{1}{80}$ to $\frac{1}{20}$. This is, of course, not the case on the higher portion of the induction curve, as is shown by the results of figs. 21 and 22, although the resultant force H is reduced by the induced currents.

In fig. 23 the maximum amperes in the copper coils is ·24, and the periodic time is reduced to 4. An inspection of these curves shows the marked effect of change of frequency, coil No. 2 being exceedingly diminished in amplitude as compared with No. 3.

As an example of the practical bearing of this portion of the paper, suppose we have a transformer core made out of iron wire, 1 millim. in diameter, the wires being perfectly insulated from one another. The outside diameter of our outer tube is 101·6 millims. Similar events will therefore happen at times, varying as $\left(\frac{1}{101\cdot6}\right)^2$. Take the case of fig. 19, in which the periodic time is 80 seconds, and the maximum average induction per sq. centim. is about 7000.

$\frac{(101\cdot6)^2}{80} = 129$ periods per second, and this is an example which might arise in practice. The ergs dissipated per cycle per cub. centim. are 3820 by induced currents, and about 3000 by magnetic hysteresis. We see further, from fig. 20, that at 500 periods per second only the outside layers of our 1 millim. wire are really useful.

As another example, take the case of an armature core of a dynamo electric machine in which a frequency of 1000 complete periods per minute might be taken.

In fig. 21 the periodic time is 80, and the maximum average induction per sq. centim. is 15,000.

We have

$$\tfrac{1}{17} = 80\ (x/101\cdot6)^2$$

$$x = 101\cdot6/36 = \text{nearly 3 millims.}$$

The ergs dissipated per cycle per cub. centim. are 26,000 by induced currents, and about 17,000 by magnetic hysteresis. This shows that according to good practice, where the wires in armature cores are of an order of 1 or 2 millims. diameter, the loss by induced currents would be but small as compared with the loss by magnetic hysteresis. This, of course, assumes the wires to be perfectly insulated from one another, which is not always realised in practice.

Both the armature cores of dynamos and the cores of transformers are now usually made of plates instead of wire; roughly speaking a plate in regard to induced currents in its substance is comparable to a wire of a diameter double the thickness of the plate. We infer that the ordinary practice of making transformer plates about $\frac{1}{2}$ millim. thick, and plates of armature cores 1 millim. thick, is not far wrong. Not much is lost by local currents in the iron, and the plates could not be much thicker without loss.*

TABLE I.

	Maximum amperes in magnetizing coils.	Maximum force in C.G.S. units. H	Maximum induction per sq. centim. B
Fig. 10	·0745	1·7	
„ 11	·138	3·0	
„ 12, Table II. .	·24	4·96	8,000
	·49	10·1	12,820
„ 13, „ III. .	·774	16·0	14,495
„ 9, „ IV. .	1·80	37·2	15,480
	2·31	47·6	
	6·5	134·5	

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* The question of dissipation of energy by local currents in iron has been discussed by Professors J. J. THOMSON and EWING. *See* the 'Electrician,' April 8th and 15th, 1892.

TABLE II.

Time in seconds after reversal.	Curve I.			Curves II. and III.				Force due to current in magnet coils. H_1.	Radius 1·22 cm.		Radius 3·18 cm.		Radius 5·08 cm.		Curve I.			Curve II.			Curve III.		
	Area in $\frac{4}{25}$ diagram squares of Curve 1. Fig. 12.	Change of Induction per sq. cm. in C.G.S. units.	Induction per sq. cm. B.	Area in $\frac{4}{25}$ diagram squares of Curve 2. Fig. 12.	Area of Curve 2, minus Curve 1, in $\frac{4}{25}$ diagram squares.	Change of induction per sq. cm. in C.G.S. units.	Induction per sq. cm. B.		E.M.F. in volts in 10^{-6}.	Amperes per sq. cm.	E.M.F. in volts in 10^{-6}.	Amperes per sq. cm.	E.M.F. in volts in 10^{-6}.	Amperes per sq. cm.	Area in 2 diagram squares. Radii 1·22 to 5·08 of Curves in Fig. 12A.	Force due to induced currents. H_2.	Resultant force. H.	Area in 2 diagram squares. Radii 3·18 to 5·08 of Curves in Fig. 12A.	Force due to induced currents. H_2.	Resultant force. H.	Area in 2 diagram squares. Radii 2·2 to 5·08 of Curves in Fig. 12A.	Force due to induced currents. H_2.	Resultant force. H.
0	0	0	−8,680	0	0	0	−7,720	−4·96	0	0	0	0	0	0	0	0	−4·96	0	0	−4·96	0	0	−4·96
2	1·5	..	..	13·7	12·2	1,842	−5,878	+4·86	36	·377	268	1·06	888	2·14	21.4	−5·3	− ·44	15·0	−3·77	+1·09	19·1	−4·8	+ ·06
3	2·7	2,130	−6,550	20·9	18·2	2,744	−4,976	+4·96	32	·335	247	·985	787	1·89	21·1	−5·28	− ·32	14·4	−3·62	+1·34	18·4	−4·6	+ ·36
7	5·1	4,024	−4,656	47·1	42	6,315	−1,405	+4·96	20	·209	310	1·23	437	1·05	18·9	−4·73	+ ·23	11·6	−2·92	+2·04	17·0	−4·2	+ ·76
11	7·1	5,604	−3,076	76·0	68·9	10,360	+2,460	+4·96	22	·230	247	·985	291	·699	14·6	−3·6	+1·16	8·4	−2·11	+2·85	12·7	−3·2	+1·76
15	10·2	8,046	− 634	98·0	87·8	13,203	+5,483	+4·96	52	·544	180	·717	204	·490	12·9	−3·2	+1·76	6·0	−1·51	+3·45	9·7	−2·4	+2·56
19	16	12,623	+3,943	111·5	95·5	14,360	+6,640	+4·96	52	·544	113	·450	144	·346	8·56	−2·14	+2·82	4·0	−1·0	+3·96	6·4	−1·6	+3·36
23	19·7	15,540	+6,860	119·5	99·8	15,006	+7,286	+4·96	24	·251	52	·207	88	.211	4·23	−1·05	+3·91	2·0	− ·50	+4·46	3·0	− ·75	+4·21
35	22	17,360	+8,680	124·7	102·7	15,440	+7,720	+4·96	0	0	0	0	0	0	0	0	+4·96	0	0	+4·96	0	0	+4·96

TABLE III.

Time in seconds after reversal.	Induction.			Radius 1·22 cm.		Radius 3·18 cm.		Radius 5·08 cm.		Force due to current in magnet winding. H_1.	Force taken from the given cyclic curve. H.	Difference of forces. H_1 and H.	Area in 4 diagram squares corresponding to differences of forces. H_1 and H.	Area in 4 diagram squares of curves in Fig. 13A.
	Area in $\frac{4}{25}$ diagram squares of Curve 1, fig. 13.	Change of induction per sq. cm. in C.G.S units.	Induction per sq. cm. B.	E.M.F. in volts in 10^{-6}.	Amp. per sq. c.m.	E.M.F. in volts in 10^{-6}.	Amp. per sq. cm.	E.M.F. in volts in 10^{-6}.	Amp. per sq. cm.					
0	0	0	−14,495	0	0	0	0	0	0	+16·02	16·0	0	0	0
1	0·92	1,814	−12,680	99	·95	902	3·48	4615	11·1	−14·09	3·0	17·0	6·7	7·9
2	1·9	3,747	−10,748	91·3	·88	1534	5·91	3077	7·4	−14·83	0	14·8	5·9	7·8
3	2·5	4,930	− 9,565	79·9	·77	1804	6·95	2460	5·9	−15·13	0·5	14·6	5·8	8·0
4	3·7	7,298	− 6,197	91·3	·88	1579	6·09	1850	4·4	−15·28	1·0	14·3	5·7	8·8
5	4·8	9,467	− 5,028	205	1·98	1331	5·13	1540	3·7	−15·43	1·2	14·2	5·6	7·8
6	10·0	19,723	+ 5,228	647	5·87	902	3·48	1080	2·6	−15·58	2·7	12·9	5·2	7·1
7	13·6	26,825	+12,330	152	1·47	541	2·09	810	1·9	−15·8	8·0	7·8	3·1	3·9
8	14·4	28,400	+13,905	26·6	·26	158	·61	310	0·7	−15·91	11·9	4·01	1·6	1·6
9	14·6	28,795	+14,300	3·8	·04	22·6	·09	77	0·2	−16·02	14·3	1·72	·7	·8
10	14·7	28,990	+14,495	0	0	0	0	0	0	−16·02	16·0	0	0	0

TABLE IV.

Time in seconds after reversal.	Induction, fig. 9B.			Radius 1·22 cm.		Radius 3·18 cm.		Radius 5·08 cm.		Area in diagram squares of Curves in fig. 9A, radii 1·22 to 5·08.	Force due to induced currents. H_2.	Force due to current in magnet winding. H_1.	Resultant force. H.
	Area in $\frac{4}{25}$ diagram squares of Curve 1, fig. 9.	Change of induction per sq. cm. in C.G.S. units.	Induction per sq. cm. B.	E.M.F. in volts in 10^{-6}.	Amperes per sq. cm.	E.M.F. in volts in 10^{-6}.	Amperes per sq. cm.	E.M.F. in volts in 10^{-6}.	Amperes per sq. cm.				
0	0	0	−15,480	0	0	0	0	0	0		0	−37·2	−37·2
1	·9	1,775	−13,705	116	1·17	1150	4·58						
2	2·2	4,340	−11,140	116	1·17	3775	15·0	4595	11·05	22·2	−55·7	+34·1	−21·6
3	3·3	6,510	− 8,970	145	1·52	2310	9·2	2872	6·9	14·2	−35·6	+34·1	− 1·5
4	9·8	19,300	+ 3,820	1160	11·7	1569	6·25	1436	3·45	14·2	−35·6	+36·4	+ .8
5	15·6	30,800	+15,320	0	0	524	2·09	718	1·72	3·2	− 8·0	+36·4	+28·4
6	15·7	30,960	+15,480	0	0	0	0	0	0	0	0	+37·2	+37·2

TABLE V.

Figures.	Tables.	Periodic time in seconds.	Force due to current in copper coils.		Max. resultant force. H.	Max. average induction per sq. cm. of Coil 1, in C.G.S. units. B.	Comparative areas of Curves Nos. 1, 2, 3, figs. 19, 20, 21, 22.			$\frac{1}{4\pi}\int H dB$ taken from EWING.	Ergs per cycle per cubic cm. dissipated by induced currents obtained from figs. 19D, 20D, 21D, 22D.
			Max. amperes.	Max. H_1.			No. 1.	No. 2.	No. 3.		
17		80	·1			2,280					545
18		20	·1			827					477
19, 19A, 19B, 19C, 19D,	VI.	80	·23	4·87	3·67	7,690	9·7	62·5	139	3,000	3,820
20, 20A, 20B, 20C, 20D,	VII.	20	·23	4·87	1·32	1,630	1·5	22·7	69·4		2,750
		80	·35			11,300					
		20	·35			2,700					
		80	·74			16,000					
		20	·74			13,700					
21, 21A, 21B, 21C, 21D,	VIII.	80	1·06	21·9	21·8	15,000	3·3	22·2	60·0	17,000	26,200
22, 22A, 22B, 22C, 22D,	IX.	20	1·06	21·9	13·0	15,500	2·1	18·5	43·4		54,500
23		4	·24								

TABLE VI.—Periodic Time, 80 seconds. Maximum Current in Copper Coils, ·23 ampere.

Time in seconds.	Coil 1.				Coil 2.				Coil 3.				Average induction per sq. cm. of Coil 1.			Current in copper coils.			Magnetizing force due to induced currents.		Resultant magnetizing force per cm. linear. H.	Energy dissipated by induced currents.	
	Deflection on D'Arsonval.	E.M.F. in 10^{-6} volts per turn.	Amperes per sq. cm.	Watts per sq. cm. in 10^{-6}.	Deflection on D'Arsonval.	E.M.F. in 10^{-6} volts per turn.	Amperes per sq. cm.	Watts per sq. cm. in 10^{-6}.	Deflection on D'Arsonval.	E.M.F. in 10^{-6} volts. per turn.	Amperes per sq. cm.	Watts per sq. cm. in 10^{-6}.	Area of E.M.F. curve in $\frac{1}{25}$ diagram squares.	Half max. area ± other areas in sq. cm.	Induction per sq. cm. in C.G.S. units. B.	Deflection on D'Arsonval.	Amperes.	Force per cm. linear. H_1.	Area in diagram squares of curves in Fig. 19A, from rad. 1·27 to 5·08.	Force per cm. linear. H_2.		Area in diagram squares of curves in Fig. 19C, from rad. 0 to 5·08.	Watts per cm. in 10^{-6}.
30	+ 0	..	..	..	+ 1	..	..	..	0	..	..	..	..	..	..	+ 67	+ ·204						
32	+ 0	..	..	..	0	..	..	..	− 1	17·5	..	..	..	..	..	+ 61	+ ·186						
34	+ 0	..	..	..	− 1	10·9	..	..	− 1·5	26·3	..	..	..	..	..	+ 53	+ ·162	+ 3·35					
			0	..	..	..	·06	..	..	..	·09	..	0	+ 19·5	7690	..	..	+ 3·1	2·5	·31	+ 3·41		
36	− 0	..	..	..	− 2	21·8	·08	1·74	− 3	52·6	·13	6·84	..	..	..	+ 45	+ ·137	+ 2·85					
38	− ·5	2·13	..	..	− 3	32·7	..	..	− 5	87·7	..	..	..	..	..	+ 34	+ ·104						
40	− 1·0	4·26	·04	·17	− 5	54·5	·21	11·4	− 20	351	·85	298	·5	+ 19·0	7496	+ 10	+ ·03	+ ·63	12·5	1·57	+ 2·2	3·9	195
42	− 2	8·51	..	..	− 10	109	..	..	− 25	439	..	..	..	..	..	− 10	− ·03						
44	− 3·5	14·9	·14	2·09	− 11	120	·46	55·2	− 30	527	1·27	669	2·15	+ 17·35	6844	− 30	− ·091	− 1·90	22	2·77	+ ·87	10·5	525
46	− 4·5	19·2	·18	3·46	− 13	142	·55	78·1	− 35	614	1·49	915	..	..	..	− 40	− ·122	..	..	..	..	16·4	820
48	− 3·5	14·9	·14	2·09	− 11·5	125	·48	60	− 30	527	1·27	669	5·8	+ 13·7	5405	− 50	− ·152	− 3·16	21	2·64	− ·52	10·5	525
50	− 3	12·8	..	..	− 14	153	..	..	− 25	439	..	..	..	..	..	− 59	− ·18						
52	− 3	12·8	·12	1·54	− 19	207	·80	165	− 25	439	1·06	465	8·2	+ 11·3	4460	− 66	− ·20	− 4·17	27·7	3·48	− ·69	14·4	720
54	− 3	12·8	..	..	− 22	240	..	..	− 23	404	..	..	..	..	..	− 70	− ·21						
56	− 3	12·8	·12	1·54	− 23	251	·97	243	− 20	351	·85	298	10·9	+ 8·6	3390	− 73	− ·22	− 4·62	29	3·65	− ·97	15	750
58	− 4	17·0	·16	2·72	− 22	240	·93	223	− 18	316	·76	240	12·2	+ 7·3	2880	− 75	− ·229	− 4·74	27·7	3·48	− 1·26	12.8	640
60	− 6	25·5	·25	6·37	− 20	218	·84	183	− 15	263	·63	166	14	+ 5·5	2170	− 77	− ·235	− 4·87	28	3·52	− 1·35	11·5	575
62	− 10	42·6	·41	17·5	− 17	185	·71	131	− 12	211	·51	108	17·7	+ 1·8	710	− 77	− ·235	− 4·87	24·9	3·13	− 1·74	7·5	375
64	− 15	63·8	·62	39·6	− 13	142	·55	78	− 10	175	·42	73·5	23	− 3·5	1380	− 77	− ·235	− 4·87	22	2·77	− 2·10	5·8	290
66	− 13	55·3	·53	29·3	− 10	109	·42	45·8	− 7	123	·3	36·9	29·4	− 9·9	3910	− 75	− ·229	− 4·74	17·5	2·20	− 2·54		
68	− 10	42·6	·41	17·5	− 7	76·3	·29	22·1	− 4	70·2	·17	11·9	34·1	− 14·6	5760	− 73	− ·22	− 4·62	13	1·63	− 2·99	1·5	75
70	− 4	17·0	·16	2·72	− 4	43·6	·17	7·4	− 2	35·1	·08	2·8	37·3	− 17·8	7020	− 68	− ·207	− 4·30	6	·75	− 3·55		
72	− 2	8·5	·08	·68	− 1	10·9	·04	·45	0	0	0	0	38·5	− 19·0	7500	− 63	− ·192	− 3·98	2·5	·31	− 3·67		
74	− ·5	2·13	·02	·04	+ ·5	5·4	·02	·11	1	17·5	·04	·7	..	..	..	− 55	− ·168	− 3·48					
			0	..	..	..	·03	..	..	..	·7	..	39·1	− 19·5	7690	..	..	− 3·2	·6	·07	− 3·3		
76	+ ·5	2·13	·02	·04	+ 1	10·9	·04	·45	2·5	43·9	·81	..	..	..	..	− 47	− ·143	− 2·97					
78	+ 1·0	4·26	..	..	+ 2	21·8	..	..	5	87·7	..	..	..	..	..	− 35							

TABLE VII.—Periodic Time, 20 seconds. Maximum Current in Copper Coils, ·23 ampere.

Time in seconds.	Coil 1.				Coil 2.				Coil 3.				Average induction per sq. cm. of Coil 1.			Current in copper coils.			Resultant force, taken from the given cyclic curve. H.	Magnetizing force due to induced currents.			Energy dissipated by induced currents.	
	Deflection on D'ARSONVAL.	E.M.F. in 10^{-6} volts per turn.	Amperes per sq. cm.	Watts per sq. cm. in 10^{-6}.	Deflection on D'ARSONVAL.	E.M.F. in 10^{-6} volts per turn.	Amperes per sq. cm.	Watts per sq. cm. in 10^{-6}.	Deflection on D'ARSONVAL.	E.M.F. in 10^{-6} volts per turn.	Amperes per sq. cm.	Watts per sq. cm. in 10^{-6}.	Area of E.M.F. curve, in $\frac{2}{125}$ diagram squares.	Half max. area ± other areas, in sq. cm.	Induction per sq. cm. in C.G.S. units. B.	Deflection on D'ARSONVAL.	Amperes.	Force per cm. linear. H_1.		H_2.	Areas required from curves in fig. 20A, in diagram squares.	Areas obtained from curves in fig. 20A, as drawn, in diagram squares.	Areas in diagram squares of curves in fig. 20c, from rad. 0 to 5·08.	Watts per cm. in 10^{-6}.
6	3	12·9	·125	..	42	350	1·35	..	16	500	1·20	..	..	..	..	−39	·235	−4·86						
8	4	17·2	·166	2·86	30	250	·965	241	− 8	250	·602	150	..	..	..	−32	·193	−3·99						
10	0	0	0	0	0	0	0	0	+10	312	·752	235	0·0	+8·25	+1630	−10	·060	−1·25	+1·32	·07	·22	·4	·3	30
11	..	..	..	..	..	..	..	..	..	..	..	..	0·8	+7·45	+1470	..	..	..	+ ·75					
12	+5·5	23·7	·229	5·43	+23	192	·74	142	+42	1310	3·16	4140	2·8	+5·45	+1075	+28	·169	+3·49	+ ·20	3·69	11·7	12·5	14·2	1420
13	..	..	..	..	..	..	..	..	..	..	..	..	5·0	+3·25	+ 641	..	..	..	− ·30					
14	+4	17·2	·166	2·86	+24	200	·772	154	+28	874	2·11	1840	6·9	+1·35	+ 266	+37	·223	+4·62	− ·45	4·17	13·3	13·0	15·0	1500
15	..	..	..	..	..	..	..	..	..	..	..	..	8·5	− ·25	− 49	..	..	..	− ·60					
16	+3·5	15·1	·145	2·19	+42	350	1·35	472	+19	593	1·43	848	10·0	−1·75	− 345	+39	·235	+4·86	− ·90	3·96	12·6	14·9	16·8	1680
17	..	..	..	..	..	..	..	..	..	..	..	..	11·7	−3·45	− 680	..	..	..	− ·95					
18	+4	17·2	·166	2·86	+35	292	1·13	330	+ 9	281	·677	190	13·3	−5·05	− 996	+33	·199	+4·12	−1·10	3·02	9·6	11·0	8·2	820
19	..	..	..	..	..	..	..	..	..	..	..	..	15·0	−6·75	−1330	..	..	..	−1·22					
20	+2	8·6	·083	·714	+ 8	66·7	·257	17·1	0	0	0	0	16·3	−8·05	−1590	+10	·060	+1·25	−1·30	1·95	6·2	4·5	·75	75
20·5	..	..	..	..	..	..	..	..	..	..	..	..	16·5	−8·25	−1630	..	..	..	−1·32					
22	−5	21·5	·208	4·47	−24	200	·772	154	42	1310	3·16	4140	..	..	..	−26	·157	−3·24						
24	−4	17·2	·166	..	25	209	·804	..	30	937	2·26	..	..	..	..	−36	·217	−4·49						
26	−3	..	·125	..	41	..	1·32	..	22	..	1·65	..	..	..	..	−39	·235	−4·86						

TABLE VIII.—Periodic Time, 80 seconds. Maximum Current in Copper Coils, 1·06 amperes.

Time in seconds.	Coil 1.				Coil 2.				Coil 3.				Average induction per sq. cm. of Coil 1.			Current in copper coils.			Magnetizing force due to induced currents.		Resultant magnetizing force per cm. linear. H.	Energy dissipated by induced currents.	
	Deflection on D'ARSONVAL.	E.M.F. in 10^{-6} volts per turn.	Amperes per sq. cm.	Watts per sq. cm. in 10^{-6}.	Deflection on D'ARSONVAL.	E.M.F. in 10^{-6} volts per turn.	Amperes per sq. cm.	Watts per sq. cm. in 10^{-6}.	Deflection on D'ARSONVAL.	E.M.F. in 10^{-6} volts per turn.	Amperes per sq. cm.	Watts per sq. cm. in 10^{-6}.	Area of E.M.F. curve in $\frac{1}{25}$ diagram squares.	Half max. area ± other areas in sq. cm.	Induction per sq. cm. in C.G.S. units. B.	Deflection on D'ARSONVAL.	Amperes.	Force per cm. linear. H_1.	Area in diagram squares of curves in Fig. 21A from rad. 1·27 to 5·08.	Force per cm. linear. H_2.		Area in diagram squares of curves in Fig. 21C from rad. 0 to 5·08.	Watts per cm. in 10^{-6}.
12																							
14																							
16	1·0	..	..	..	4	..	..	..	3														
18	·5	..	..	..	1	..	..	..	1·5														
20	0	..	..	..	0	0	0	..	·5														
22	0	..	..	..	0	0	0	..	0														
24	0	0	0	0	0	0	0	0	0	0	0	0	..	..	..	49	1·04	+21·5	0	0	−21·5	0	0
26	0	0	0	..	.5	8·78	·0338	..	·5	22·1	·0532	..	..	..	..	45	0·95	+19·8					
28	0	0	0	..	·5	8·78	·034	..	1	44·1	·106	..	..	..	..	39	·83	+17·1					
30	0	0	0	..	1·0	17·6	·068	..	2	88·3	·213	..	0	..	..	35	·74	+15·4					
32	0	0	0	..	1·0	17·6	·068	..	2	88·3	·213	..	0	7·6	15,000	30	·64	+13·2					
34	·5	4·12	·0398	·164	2	35·1	·135	..	4	177	·426	..	0	7·6	15,000	25	·53	+11·0					
36	·5	4·12	·0398	·164	3	52·7	·203	10·7	5	221	·532	118	·1	7·5	14,800	20	·424	+ 8·78	5	+ 1·6	−10·4	·5	125
38	1·0	8·25	·0795	..	4	70·2	·271	..	8	353	·851	..	·2	7·4	14,600	12	·254	+ 5·27					
40	1·5	12·4	·119	..	8	140	·542	..	14	618	1·49	..	·55	7·05	13,900	+ 2	·042	+ ·88					
42	2·0	16·5	·159	2·62	12	211	·812	171	20	883	2·13	1880	·9	6·7	13,200	− 6	·127	− 2·63	13·8	+ 4·34	− 1·71	6·4	1,600
44	4·0	33·0	·318	10·5	15	263	1·02	268	30	1320	3·19	4210	1·3	6·3	12,400	14	·297	− 6·15				12·8	3,200
46	5·0	41·2	·398	16·4	15	263	1·02	268	40	1770	4·26	7540	2·1	5·5	10,800	20	·424	− 8·78	23·5	+ 7·39	+ 1·39	20·3	5,070
48	4·0	33	·318	10·5	45	790	3·05	2410	38	1680	4·04	6790	2·8	4·8	9,470	26	·55	−11·4	..	..	..	41·5	10,400
50	4·5	37·1	·358	13·3	51	896	3·45	3090	33	1460	3·51	5120	3·5	4·1	8,090	30	·64	−13·2	..	..	..	42	10,500
52	10	82·5	·795	65·6	45	790	3·05	2410	28	1240	2·98	3690	4·6	+3·0	+ 5,920	35	·74	−15·4	42·0	+13·2	+ 2·2	35	8,750
54	47	388	3·74	1451	35	615	2·37	1460	20	883	2·13	1880	9·65	−2·05	− 4,040	39	·83	−17·1	45·0	+14·2	+ 2·9	31·5	7,880
56	5·0	41·2	·398	16·4	10	176	·677	119	14	618	1·49	921	14·8	7·2	14,200	44	·93	−19·3	10·0	+ 3·1	+17·2	2·5	625
58	·5	4·12	·0398	·16	2	35·1	·135	4·74	4	177	·426	75	15·1	7·5	14,800	48	1·02	−21·1	3·5	+ 1·1	+20·0		
60	·5	4·12	·0398	·16	1	17·6	·068	1·20	2	88·3	·213	18·8	15·18	7·58	14,950	49	1·04	−21·5	1·6	+ ·5	+21		
62	0	0	0	..	0	0	0	..	·5	22·1	·053	·12	15·2	7·6	15,000	50	1·06	−21·9	·15	+ ·05	+21·8		
64	0	0	0	0	0	0	0	0	0	0	0	0	15·2	7·6	15,000	48	1·02	−21·1	0	0	..	0	0
66	0	0	0	..	0	0	0	..	0	0	0	..	..	..	..	43	·91	−18·9					
68	0	0	0	..	1	17·6	·068	..	0	0	0	..	..	..	..	40							
70	..	..	..	..	..	..	..	..	1·5	..	..	..	..	..	..	35							

TABLE IX.—Periodic Time, 20 seconds. Maximum Current in Copper Coils, 1·06 amperes.

Time in seconds.	Coil 1.				Coil 2.				Coil 3.				Average induction per sq. cm. of Coil 1.			Current in copper coils.			Magnetizing force due to induced currents.		Resultant magnetizing force per cm. linear. H.	Energy dissipated by induced currents.	
	Deflection on D'ARSONVAL.	E.M.F. in 10^{-6} volts per turn.	Amperes per. sq. cm.	Watts per sq. cm. in 10^{-6}.	Deflection on D'ARSONVAL.	E.M.F. in 10^{-6} volts per turn.	Amperes per sq. cm.	Watts per sq. cm. in 10^{-6}.	Deflection on D'ARSONVAL.	E.M.F. in 10^{-6} volts per turn.	Amperes per sq. cm.	Watts per sq. cm. in 10^{-6}.	Area of E.M.F. curve in $\frac{1}{5}$ diagram squares.	Half max. area ± other areas in sq. cm.	Induction per sq. cm. in C.G.S. units. B.	Deflection on D'ARSONVAL.	Amperes.	Force per cm. linear. H_1.	Area in diagram squares of curves in Fig. 22A from rad. 1·27 to 5·08.	Force per cm. linear. H_2.		Area in diagram squares of curves in Fig. 22C from rad. 0 to 5·08.	Watts per cm. in 10^{-6}.
26	10	109	1·05	..	23	1630	6·28	..	14	1920	4·63	..	..	..	..	39							
28	50	545	5·25	..	+10	708	2·73	..	+ 3	412	·992	..	..	..	..	30	·86	17·8					
30	0	0	0	0	− 2	142	·546	77·5	− 8	1098	2·64	2,900	0	+7·85	+15,500	+15	·430	+ 8·91	6·5	4·09	13·0	·5	1,250
31	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..								
32	5	54·5	·525	28·6	7	495	1·91	945	30	4120	9·92	40,900	·5	+7·35	+14,500	− 5	·143	− 2·97	22	13·8	10·0	8·0	20,000
33	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..								
34	4	43·6	·420	18·3	25	1770	6·82	12,100	24	3290	7·93	26,100	1·5	+6·35	+12,500	25	·716	14·8	38	23·9	9·1	17·6	44,000
35	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..								
36	10	109	1·05	114	25	1770	6·82	12,100	15	2060	4·96	10,200	2·8	+5·05	+ 9,960	37	1·06	22·0	41	25·8	+ 3·8	13·5	33,700
37	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..								
38	50	545	5·25	2860	10	708	2·73	1930	− 5	686	1·65	1,130	9·4	−1·55	− 3,060	30	·86	17·8	+25	15·7	− 2·1	4·8	12,000
39	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..								
40	0	0	0	0	0	0	0	0	+ 5	686	1·65	1,130	15·7	−7·85	−15,500	−15	·43	8·91	− 1·5	·94	9·8	·07	175
42	5	54·5	·525	..	5	354	1·36	..	30	4120	9·92	..	..	..	..	+10	·287	5·94	..				
44	4	43·6	·420	..	7	495	1·91	..	22	3020	7·27	..	..	..	..	30	·86	17·8	..				